FIRST LESSONS

IN

MENTAL AND WRITTEN

ARITHMETIC;

UPON THE INDUCTIVE PLAN.

DEVELOPING

THE FUNDAMENTAL RULES, AND APPLYING THEM TO A GREAT VARIETY OF PRACTICAL EXAMPLES.

BY ORLANDO BLANCHARD, A. M.

AUTHOR OF PHONETICAL AND PHONETICAL ARITHMETIC.

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PREFACE.

THIS work is designed to furnish the younger classes of pupils with a suitable course of mental exercises on numbers, and to initiate them into the methods of operating in the fundamental rules, before they commence a general course of written arithmetic. The exercises are progressive. The pupil is first taught to count as far as ten, and to add and subtract, as far as possible, with numbers not exceeding ten, after which he is led by steps sufficiently gradual, to the use of larger numbers. Sudden transitions from counting thumbs to the solution of complex questions, are carefully avoided. The opposite extreme, of stopping within the limits of what every child knows without instruction, is also avoided. While nothing is designedly attempted that children cannot understand, and enjoy as an agreeable exercise, yet the operations are so extended as to be useful for older pupils, and even for men of business. In this work, all is not left to the untaught ingenuity of every one who may study it. The author has given the reasoning process on each class of questions, and generally pursued the investigation to a conclusion or rule to guide in similar cases. He entertains the hope that teachers will find in this work an acceptable addition to their means of instruction in the first principles of numbers.

O. B.

CAZENOVIA, March, 1854.

THREE TIMES FOUR.

Are they 12? or, *is it* 12? Custom being divided on this grammatical question, I think it consistent and correct to say, *it is* 12. If the plural verb *are* is required, the plural pronoun *they* is proper, and we may say, 2 *times* 3 *are* how many? *They are* 6. If John says, 6 times 12 *are* 74, Peter may say, *Are they* 74? No. *they are* 72. If the word *times* requires *are*, then 3 times John's age *are* equal to his father's age; and 4 times the *number* of sheep in a certain flock, *are* 200. If it is proper to ask, "5 less 3 *are* how many?" the proper answer is, *they are* 2. If I should ask, how many *are* a dozen, I must admit the answer, *they are* 12! But as we reject *they*, in these examples, so we must reject *are*. When we say, 3 times 4 trees are 12 trees, we have reference to the *objects* counted; but in saying 3 times 4 *is* 12, we mean, that 3 times the *number* 4, *is* the *number* 12. Here we use 4 and 12, not as numeral *adjectives*, but as *nouns*, the *names* of particular *numbers*, and as such, each conveys the idea of *unity*, and the entire expression is the subject of *is*, and conveys the idea of unity. If we use *are* in multiplication, we ought to use the same in division, and say 3 *are* in 12, 4 times; but almost everybody says, 3 *is* in 12, 4 times. The old authors, English and American, say *is* in multiplication. See Ward, Dilworth, and Pike, and some very respectable present authors; Hutton, De Morgan, Mr. Todd, and last, not least, Dr. Brewster's Edinburgh Encyclopedia, says "5 times 6 is 30."

Some of our most respectable seminaries have lately considered this question, and decided in favor of *is*.

When an *abstract number* is mentioned, we consider the collection as single, and say that *it is;* but when the *individuals*, or particular *objects* that compose the collection, are considered, we properly say that *they are*.

But, in addition, we properly say, 2 and 2 *are* 4, because we have two nominatives connected by *and*, requiring a plural verb, although each separately conveys the idea of unity.

ARITHMETIC.

CHAPTER I.

INTRODUCTION.

SECTION I.

OF NUMBERS NOT EXCEEDING TEN.

1. How many fingers, (including the thumb,) have you on one hand?

2. How many have you on both hands? Count them.

One, Two, Three, Four, Five.
Six, Seven, Eight, Nine, Ten.

3. How many fingers have you on each hand?

4. Then, five and five are how many?

Rest the ends of all your fingers, (and thumbs,) on the table or desk before you.

5. Now raise two fingers of the right hand, and one of the left; how many fingers have you raised?

Then, two and one are how many?

6. Hold up two fingers of the right hand, and two of the left; how many fingers do you hold up?

Then, two and two are how many?

7. Hold up three fingers of the right hand, and two of the left; three and two are how many?

8. Hold up four of the right hand, and two of the left; four and two are how many?

9. Hold up four of the right hand, and three of the left; four and three are how many?

10. Hold up four of the right hand, and four of the left; four and four are how many?

11. Five of the right hand, and one of the left, are how many?

12. Five and two are how many? Five and three? Five and four? Five and five?

13. One and one are how many? One and two? One and three? One and four? One and five? One and six? One and seven? One and eight? One and nine?

14. Two and one are how many? Two and two? Two and three? Two and four? Two and five? Two and seven? Two and eight?

15. Three and one are how many? Three and two? Three and three? Three and four? Three and five? Three and six? Three and seven?

16. Four and two are how many? Four and one? Four and three. Four and five? Four and four? Four and six?

17. Five and one are how many? Five and three? Five and two? Five and four? Five and five?

18. Six and one are how many? Six and three? Six and two? Six and four?

19. Seven and one are how many? Seven and three? Seven and two?

20. Eight and two are how many? Eight and one?

21. Nine and one are how many?

22. One, two and three are how many?

23. Two, three and four are how many?

SECTION II.

1. Rest the five fingers of your right hand on the table, then raise one of them; how many remain on the table? One from five leaves how many? One and how many are five?

2. Raise two of the five fingers of the right hand, and how many remain on the table? Two from five leaves how many? Two and how many are five?

3. Hold up three fingers of the right hand; how many remain? Three and how many are five?

4. Four from five leaves how many?

5. Rest your ten fingers on the table, then raise one of them; how many remain? One from ten leaves how many?

6. Raise two fingers, and how many remain? Two from ten leaves how many?

7. Raise three, and how many remain? Three from ten, how many? Four from ten? Five from ten? Six from ten? Seven from ten? Eight from ten? Nine from ten? Ten from ten?

8. Three trees are on one side of the walk, and two on the other; how many are on both sides?

9. Five trees are by the walk, three are on the right hand, and the others on the left; how many are on the left? Three from five leaves how many? Two from five leaves how many?

10. Four houses are seen on the right hand side of the street, and three on the left; how many on both sides?

11. Seven houses are seen on one street; four on the right hand, and the others on the left; how many on the left? Four from seven leaves how many?

12. Five pitchers are on the counter and four on a shelf; how many pitchers in all?

13. Nine pitchers were on the shelf, and five of them are taken down; how many remain? Five from nine leaves how many? Four from nine how many?

14. Eight boys came to school, five of them wearing hats, and the rest caps; how many wore caps? Five from eight how many? Three from eight how many?

15. Mary has five roses, and Jane five; how many have they both?

16. Sarah and Helen have ten needles; six of them are Sarah's; how many are Helen's?

17. How many are five and three? Six and two? Four and four? Seven and one?

18. One from eight leaves how many? Two from eight? Three from eight? Four from eight? Five from eight? Seven from eight?

19. How many are five and four? Six and three? Seven and two? Eight and one?

20. One from nine leaves how many? Two from nine? Three from nine? Four from nine? Six from nine? Five from nine? Seven from nine? Eight from nine?

21. How many are five and five? Six and four? Seven and three? Eight and two? Nine and one?

22. How many is ten less one? Ten less three? Ten less two? Ten less four? Ten less six? Ten less five? Ten less seven? Ten less nine? Ten less eight?

23. William gave four cents for an orange, two cents for a pear, and one cent for an apple; how much did they all cost?

24. Mary has two books, Jane three, and Ruth four; how many have they all?

25. William having ten cents, gave three cents to a beggar, and paid four cents for a toy book; how many cents had he remaining?

26. James went ten miles; he rode with a friend two miles, in a stage four miles, and walked the rest of the way: how far did he walk?

27. A post ten feet long, is two feet in the mud, three feet in the water, and the rest of its length above the water; what is the length above the water?

28. If you earn six dollars, and pay two dollars for a hat, and one dollar for a pair of shoes, how much money will you have left?

29. Henry bought seven apples and three peaches; he gave four apples and one peach to James; how many pieces of fruit has he remaining?

30. A stage set out with five passengers; three passengers left, and two others took their places; how many were then in the stage?

31. James had seven apples; he gave three to Henry, and two to George; how many has he remaining?

32. Sarah bought ten needles; she has broken three of them, and lost two; how many remain?

33. Mary was yesterday the fourth in her class; to-day she has gained two places, and lost three; what is her place now? Six less three is what number?

34. Henry bought three oranges and four peaches; he has eaten one orange and two peaches, and given one orange to James; how many pieces of fruit has he left? How many of each kind has he left?

SECTION III.

OF NUMBERS EXCEEDING TEN.

How many stars in these two columns of ten each?

One *	One and ten *	Eleven.
Two *	Two and ten *	Twelve.
Three *	Three and ten *	Thirteen.
Four *	Four and ten *	Fourteen.
Five *	Five and ten *	Fifteen.
Six *	Six and ten *	Sixteen.
Seven *	Seven and ten *	Seventeen.
Eight *	Eight and ten *	Eighteen.
Nine *	Nine and ten *	Nineteen.
Ten *	Ten and ten *	Twenty.

Thus, it is as easy to count two tens, as to count one ten, since we only repeat the first ten to make the second. The middle column of words, one and ten, two and ten, &c., shows the primitive method of counting, as we find it in the Greek and Latin languages, except that the *and* is omitted; but in English we have adopted eleven and twelve, for one ten, two ten, and have changed three ten, four ten, &c., to thirteen, fourteen, &c., which are words of more agreeable sound. The pupil may first be taught the names in the middle column, and those in the third afterwards.

Some nations count onward from twenty by saying, one and twenty, two and twenty, &c.; but in English we say, twenty one, twenty two, twenty three, meaning, two tens one, two tens two, two tens three, &c., to twenty nine. Next is thirty, meaning three tens; then thirty one, thirty two, &c., to thirty nine, forty, (four tens.) Next forty one, forty two, &c., to fifty, (five tens,) &c.

Sixty is six tens, seventy is seven tens, eighty is eight tens, ninety is nine tens, one hundred is ten

tens. Then we repeat, thus: one hundred one, one hundred two, &c., to nine hundred ninety nine, one thousand, which means ten hundreds, or one hundred tens.

One thousand thousands is called a million; a thousand millions, a billion; a thousand billions, a trillion. Thus we reckon great numbers by repeating ten, till there are ten tens, making a hundred; then we repeat the hundreds till there are ten hundreds making a thousand. A unit is a single thing of any kind whatever; as, one apple, one house, one tree.

Ten units make one ten.
Ten tens make one hundred.
Ten hundreds make one thousand.
One thousand thousands make one million.

SECTION IV.

NUMBERS EXPRESSED BY FIGURES.

Numbers may be written in words, but we have a shorter method of writing them in figures; and as we count all numbers by units, tens, hundreds, &c., so we write all numbers with only ten figures, called the Arabic figures, from their reputed inventors, the Arabians. These figures are,

0	1	2	3	4	5	6	7	8	9
Naught.	One.	Two.	Three.	Four.	Five.	Six.	Seven.	Eight.	Nine.

The 0, when alone, signifies nothing; it is often called a cipher. The others are significant figures or digits. Any number of units less than ten is written with one figure, as seen above.

We write ten with 1 and 0, thus 10; eleven, 11;

twelve, 12; and all numbers between nine and one hundred, with two figures, tens and units; the left hand figure expressing the tens, and the right hand figure the units, as in the following

TABLE.

1 One	35 Thirty-five	68 Sixty-eight
2 Two	36 Thirty-six	69 Sixty-nine
3 Three	37 Thirty-seven	70 Seventy
4 Four	38 Thirty-eight	71 Seventy-one
5 Five	39 Thirty-nine	72 Seventy-two
6 Six	40 Forty	73 Seventy-three
7 Seven	41 Forty-one	74 Seventy-four
8 Eight	42 Forty-two	75 Seventy-five
9 Nine	43 Forty-three	76 Seventy-six
10 Ten	44 Forty-four	77 Seventy-seven
11 Eleven	45 Forty-five	78 Seventy-eight
12 Twelve	46 Forty-six	79 Seventy-nine
13 Thirteen	47 Forty-seven	80 Eighty
14 Fourteen	48 Forty-eight	81 Eighty-one
15 Fifteen	49 Forty-nine	82 Eighty-two
16 Sixteen	50 Fifty	83 Eighty-three
17 Seventeen	51 Fifty-one	84 Eighty-four
18 Eighteen	52 Fifty-two	85 Eighty-five
19 Nineteen	53 Fifty-three	86 Eighty-six
20 Twenty	54 Fifty-four	87 Eighty-seven
21 Twenty-one	55 Fifty-five	88 Eighty-eight
22 Twenty-two	56 Fifty-six	89 Eighty nine
23 Twenty-three	57 Fifty-seven	90 Ninety
24 Twenty-four	58 Fifty-eight	91 Ninety-one
25 Twenty-five	59 Fifty-nine	92 Ninety-two
26 Twenty-six	60 Sixty	93 Ninety-three
27 Twenty-seven	61 Sixty-one	94 Ninety-four
28 Twenty-eight	62 Sixty-two	95 Ninety-five
29 Twenty-nine	63 Sixty-three	96 Ninety-six
30 Thirty	64 Sixty-four	97 Ninety-seven
31 Thirty-one	65 Sixty-five	98 Ninety-eight
32 Thirty-two	66 Sixty-six	99 Ninety-nine
33 Thirty-three	67 Sixty-seven	100 One hundred
34 Thirty-four		

The art of writing numbers is called *notation*.

The art of reading numbers is called *numeration*.

The teacher can teach a young class the nature of the numerical scale, the art of counting, and the principles of notation and numeration, thus;

HUNDREDS.	TENS.	UNITS.
p p p p p	b b b b b	c c c c c
p p p p p	b b b b b	c c c c c

Let each pupil have a slate before him, marked with the words, Hundreds, Tens, Units, as in the foregoing diagram; and on it ten corns in c c, &c., for the units; ten beans in b b, &c., for the tens; and several pins or pebbles in p p, &c., for the hundreds.

Then as the teacher counts deliberately, one, two, three, &c., let each pupil move forward a corn for each unit counted; and when the ten corns are all moved forward, take them back, and move forward one bean for a ten. Then use the ten units again in the same manner, and when twenty are counted, move forward another bean, &c., to ninety nine, nine tens and nine units. Then at the next unit counted, move forward the tenth unit; take back the ten corns, and move forward the last bean; then take back the ten beans, and move forward one hundred. Now begin again as at first.

Let the counting be frequently stopped, and the pupils be required to write, in tens' place, the number of tens counted, and in units' place the units in addition to the tens. Thus notation will be taught at the same time. This method is better than a "Numerical Frame," because here the pupils are *employed*, instead of being mere spectators.

Read the following numbers:

10	12	11	13	14	16	19	15	16
17	21	30	18	27	22	29	28	26
57	95	85	75	60	36	46	65	66
112	204	209	872	278	485	584	376	365
907	970	385	358	583	853	671	617	167
324	842	248	284	482	428	609	906	960

The numbers in the lower line are, Three hundred twenty-four, Eight hundred forty-two, Two hundred forty-eight, &c.

NOTE.—It is well to dispense with the word *and* after hundred, to avoid ambiguity in reading fractions; $200\frac{7}{100}$ is two hundred, and seven one thousandths; but $\frac{207}{1000}$ is two hundred seven one thousandths.

Write the following numbers: Eighteen, Eighty-one, Fifty-seven, Seventy-five, Forty-six, Ninety-five, One hundred twenty-five, One hundred fifty-two, Seventy-eight, Six hundred forty-three, Seven hundred eighty, Seven hundred four. [704, write 0 in any place at the right, or between other figures, where no value is mentioned.] Eight hundred nine, Six hundred twenty, Nine hundred forty, Six hundred, Four hundred, Seven hundred twenty-five.

Five hundred twelve, Four hundred ten, Six hundred eleven, Two thousand one hundred fourteen, Five thousand two hundred seventy-five, One thousand three hundred eighty five, Seven thousand and eight, Seven thousand and eighty, Seven thousand and eighty-four, Eight thousand and fifteen, Five thousand one hundred eight, Five thousand one hundred eighty, Five thousand eight hundred one, Five thousand eight hundred ten, One thousand eight hundred five, Eight thousand and fifty-one, Eight thousand five hundred one, Eight thousand five hundred ten.

Nine thousand and three, Nine thousand and thirty, Nine thousand three hundred, Three thousand and nine, Three thousand and ninety, Three thousand nine hundred, Four thousand three hundred twenty-one, One thousand two hundred thirty-four, Two thousand one hundred thirty-two.

NUMERATION TABLE.

Hundreds of Quadrillions	Tens of Quadrillions	Units of Quadrillions	Hundreds of Trillions	Tens of Trillions	Units of Trillions	Hundreds of Billions	Tens of Billions	Units of Billions	Hundreds of Millions	Tens of Millions	Units of Millions	Hundreds of Thousands	Tens of Thousands	Units of Thousands	Hundreds of Units	Tens of Units	Units of Units
																8	5
															1	2	6
														3	3	0	5
													5	7	8	0	2
												3	2	8	7	0	6
											1	5	0	0	1	0	4
										2	0	7	0	3	0	0	9
									8	2	5	0	7	0	6	0	2
								3	2	7	6	8	0	1	7	2	8
							6	5	0	0	1	0	0	1	0	0	1
						8	1	0	3	0	0	4	2	8	3	0	2
				6	2	9	3	1	4	7	2	8	6	3	5	7	0	0
6	0	0	0	9	0	8	2	0	0	5	0	0	3	0	6	7	4
7	8	5	2	4	1	7	8	4	3	0	0	5	0	1	6	0	0

The orders are divided into periods of three figures each, each period having its distinctive name; the hundreds, tens and units are repeated in each period.

THE ROMAN METHOD OF NOTATION.

The Roman method of notation employs seven letters of the alphabet; thus

I.	V.	X.	L.	C.	D.	M.
1	5	10	50	100	500	1000

A letter of less value placed before one of a greater, is to be taken from the greater, but if placed after, it is to be added; as, IV., four, VI., six.

TABLE.

I.	1	X.	10	XVIII.	18	LXXX.	80
II.	2	XI.	11	XIX.	19	XC.	90
III.	3	XII.	12	XX.	20	C.	100
IV.	4	XIII.	13	XXX.	30	D.	500
V.	5	XIV.	14	XL.	40	DC.	600
VI.	6	XV.	15	L.	50	M.	1,000
VII.	7	XVI.	16	LX.	60	MM.	2,000
VIII.	8	XVII.	17	LXX.	70	MMM.	3,000
IX.	9						

CHAPTER II.

ADDITION.

SECTION I.

Addition is the process of uniting two or more numbers, to form one number; as, 2, 3, and 4, are 9. The result of addition is called the *sum* of the numbers, or the *amount*.

NOTE.—Never use the word *sum* to signify a question or an example. We solve questions and reduce examples; but we never *do sums*. What is the sum of 5 apples, 6 apples, and 7 apples. *Ans.* 18 apples.

The upright cross (+) is the sign of addition.

Two horizontal lines (=) are the sign of equality; as, 2+3=5; two and three are 5; or two plus three equals five. To add figures whose sum exceeds ten, is to find *how much* it exceeds ten; as, 8 and 6 are how many, means 8 and 6 are *ten* and *how many?*

Illustration, ❁❁❁❁❁❁❁❁** ****
8 and 6 are 10 and 4, or 14. *Ans.*

NOTE.—The illustrations in this section are intended to show that addition consists in taking numbers which are presented in promiscuous masses, and arranging them in the decimal scale; thus, if we counted by 8s, then 8 and 6 are 8 and 6. There would be no simpler expression for their sum. If we counted by 9s, then 8 and 6 are 9 and 5; or if by sixes, then 8 and 6 are 2 sixes and 2.

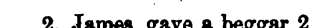
1. Thus we see that 8 and 6, are the same as 10 and 4. Hence 8 and 6 are 14 (fourteen).

2. James gave a beggar 2 cents, and John gave him 9; how many cents did they both give him?

2 and 9 are how many?

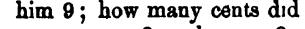
2 and 9 are 10 and 1. *Ans.* 11.

3. William walked 3 miles and rode 8; how far did he go?



3 and 8 are how many?

4. Julia paid 3 cents for tape and 9 cents for ribbon; how many cents for both?

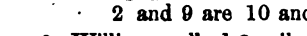


3 and 9 are how many?

5. Charles had 4 apples and William gave him 7 more; how many had he then?

4 and 7 are how many?

6. Mary had 4 needles, and her sister gave her 8 more; how many had she then?



4 and 8 are how many?

7. Four birds are on the bush, and 9 are on the ground; how many in all?

4 and 9 are how many?

8. Reuben found 5 eggs in one nest, and 6 in another; how many in both nests?

5 and 6 are how many?

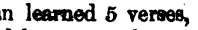
9. Amos gave 5 cents for paper, and 7 cents for a book; how many cents for both?

5 and 7 are how many?

10. Susan learned 5 verses, while Jane learned 8 both learned how many?

5 and 8 are how many?

11. James gave me 5 plums, and had 9 left; how many had he at first?



5 and 9 are how many?

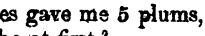
12. Samuel was 6 years old, five years ago; how old is he now?

6 and 5 are how many?

13. Edwin has 6 cents, and Seth has 6; how many have both?

6 and 6 are how many?

14. Eliza reads 6 pages of history in the morning and 7 pages in the evening; how many pages a day



6 and 7 are how many?

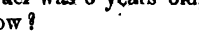
15. James caught 6 fish and Moses 8; how many did they both catch?

6 and 8 are how many?

16. If your class contains 6 boys and 9 girls, how many are in the class?

6 and 9 are how many?

17. If one book cost 7 shillings, and another 4 how much did they both cost?



7 and 4 are how many?

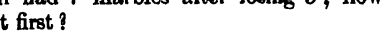
18. Eli had 7 marbles after losing 5; how many had he at first?

7 and 5 are how many?

19. John gave 7 apples for a book, and 6 for a pencil; how many apples for both?

7 and 6 are how many?

20. Henry has 7 marbles, and James 7; how many have both?

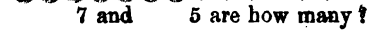


7 and 7 are how many?

21. Mary has 7 pinks, and Martha 8; how many have both?

7 and 8 are how many?

22. James has 7 oranges in one basket and 9 in another; how many in both?

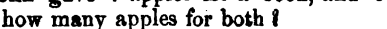


7 and 9 are how many?

23. If you go from this place north 8 miles, and John goes south 3 miles, how far apart will you be?

8 and 3 are how many?

24. Peter paid 8 cents for a ball, and gained 4 cents in the sale of it; how much did he receive for it?



8 and 4 are how many?

25. Lydia has 8 peaches, and Sophia has 5; how many have both?

8 and 5 are how many?

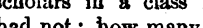
26. Mary paid 8 cents for a slate, and 6 cents for pens; how many cents for slate and pens?

8 and 6 are how many?

27. Eight scholars in a class had learned their lesson, and 7 had not; how many were in the class?

8 and 7 are how many?

28. Charles paid 8 cents for strawberries, and 8 cents for raisins; how much for both?



8 and 8 are how many?

29. Bought a barrel of flour for 8 dollars, and a hundred pounds of sugar for 9 dollars; how much did both cost?

8 and 9 are how many?

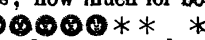
30. James paid 9 cents for a book, and had 2 cents left; how many cents had he at first?

9 and 2 are how many?

31. James gave away 9 apples and had 3 left; how many had he at first?

9 and 3 are how many?

32. Charles paid 9 cents for a writing book, and four cents for pens; how many cents for book and pens?



9 and 4 are how many?

33. Peter set 9 trees, and Robert 5; how many did both set?

9 and 5 are how many?

34. A boy paid 9 cents for a ball, and 6 cents for a top; how much for both?

9 and 6 are how many?

35. Henry took 9 peaches from one tree, and 7 from another; how many from both?

9 and 7 are how many?

36. The first story of a house is 9 feet high, and the second 8; how high are the two stories?

9 and 8 are how many?

37. John paid 9 cents for a book, and 9 cents for a slate; how much for both?

9 and 9 are how many?

Commit to memory the foregoing results, thus;

1 and 9 are 10	5 and 5 are 10
2 and 8 are 10	5 and 6 are 10 and 1
2 and 9 are 10 and 1	5 and 7 are 10 and 2
3 and 7 are 10	5 and 8 are 10 and 3
3 and 8 are 10 and 1	5 and 9 are 10 and 4
3 and 9 are 10 and 2	6 and 4 are 10
4 and 6 are 10	6 and 5 are 10 and 1
4 and 7 are 10 and 1	6 and 6 are 10 and 2
4 and 8 are 10 and 2	6 and 7 are 10 and 3
4 and 9 are 10 and 3	6 and 8 are 10 and 4

6 and 9 are 10 and 5
7 and 3 are 10
7 and 4 are 10 and 1
7 and 5 are 10 and 2
7 and 6 are 10 and 3
7 and 7 are 10 and 4
7 and 8 are 10 and 5
7 and 9 are 10 and 6
8 and 2 are 10
8 and 3 are 10 and 1
8 and 4 are 10 and 2
8 and 5 are 10 and 3
8 and 6 are 10 and 4
8 and 7 are 10 and 5
8 and 8 are 10 and 6
8 and 9 are 10 and 7
9 and 1 are 10
9 and 2 are 10 and 1
9 and 3 are 10 and 2
9 and 4 are 10 and 3
9 and 5 are 10 and 4
9 and 6 are 10 and 5
9 and 7 are 10 and 6
9 and 8 are 10 and 7
9 and 9 are 10 and 8

SECTION II.

Commit to memory the following table. Read, two and naught are two; two and one are three, &c. The class should practise upon it till they are perfectly familiar with it.

ADDITION TABLE.

2+0= 2	3+0= 3	4+0= 4
2+1= 3	3+1= 4	4+1= 5
2+2= 4	3+2= 5	4+2= 6
2+3= 5	3+3= 6	4+3= 7
2+4= 6	3+4= 7	4+4= 8
2+5= 7	3+5= 8	4+5= 9,
2+6= 8	3+6= 9	4+6=10
2+7= 9	3+7=10	4+7=11
2+8=10	3+8=11	4+8=12
2+9=11	3+9=12	4+9=13

5+0= 5	6+0= 6	7+0= 7
5+1= 6	6+1= 7	7+1= 8
5+2= 7	6+2= 8	7+2= 9
5+3= 8	6+3= 9	7+3=10
5+4= 9	6+4=10	7+4=11
5+5=10	6+5=11	7+5=12
5+6=11	6+6=12	7+6=13
5+7=12	6+7=13	7+7=14
5+8=13	6+8=14	7+8=15
5+9=14	6+9=15	7+9=16

8+0= 8	9+0= 9
8+1= 9	9+1=10
8+2=10	9+2=11
8+3=11	9+3=12
8+4=12	9+4=13
8+5=13	9+5=14
8+6=14	9+6=15
8+7=15	9+7=16
8+8=16	9+8=17
8+9=17	9+9=18

Even numbers are such as end with 0, 2, 4, 6, 8.

Odd numbers end with 1, 3, 5, 7, 9.

In adding, observe that the sum of *even* numbers is always *even*; as, 8 and 4 are 12.

The sum of *two odd* numbers is *even*; 7+3=10.

The sum of an *even* and an *odd* number is *odd*; as, 6+3=9.

Any sum containing an *odd number of odd numbers* is *odd*; all other sums are *even*.

When two numbers, whose unit figure is the same, are equally increased, the unit figure in the sums will be the same; as 8+7=15; 18+7=25; 58+7=65.

Adding 9 is the same as adding 10 and taking

away 1; thus the digit to which 9 is added is diminished by 1, and the next increased by 1; as 6+9=15; 28+9=37.

Adding 8 to any digit greater than 1, will diminish that digit by 2, and increase the next by 1; as 8+2=10; 8+25=33; 8+79=87.

Adding 7 to any digit greater than 2 diminishes it by 3, and increases the next by 1; as 7+3=10; 7+16=23.

1. Add 4+7; 14+7; 24+7.
2. Add 5+6; 15+6; 35+6; 55+6.
3. Add 3+9; 13+9; 23+9; 33+9.
4. Add 4+9; 17+9; 27+9; 25+9.
5. Add 5+8; 15+8; 17+8; 27+8.
6. Add 5+7; 16+7; 17+7; 29+7.
7. Add 8+7; 24+7; 35+7; 45+7.
8. Add 6+6; 16+6; 26+6; 36+6.

SECTION III.

COMBINATIONS OF 1, 2, 3, AND 4; THREE AT A TIME.

1. How many are 1+2+2? 1+2+3? 1+2+4?
2. How many are 1+3+4? 2+3+4? 1+1+4?
3. How many are 2+2+3? 2+2+4? 2+3+3?
4. How many are 2+2+2? 2+4+4? 3+3+4?
5. How many are 3+3+3? 3+3+1? 4+4+4?
6. How many cents have James, Peter, and John, if James has 1, Peter 3, and John 4?
7. " " " 2, " 4, " " 3?
8. " " " 3, " 4, " " 3?
9. " " " 2, " 3, " " 3?
10. " " " 3, " 3, " " 2?

11. How many needles have Mary, Jane, and Ruth, if Mary has 4, Jane 4, and Ruth 3?
12. " " " 4, " 4, " " 1?
13. " " " 3, " 3, " " 1?
14. " " " 2, " 2, " " 3?
15. " " " 4, " 4, " " 4?

SECTION IV.

COMBINATIONS OF 1, 2, 3, 4, AND 5; THREE AT A TIME.

1. How many are 1+2+5? 1+3+5? 1+4+5?
2. How many are 2+2+5? 2+3+5? 2+4+5?
3. How many are 3+2+5? 3+4+5? 3+3+5?
4. How many are 4+4+5? 4+5+5? 5+5+5?
5. If a chair cost 2 dollars, a table 3 dollars, and a bedstead 5 dollars, how much do they all cost?
6. James found 5 eggs in one nest, 2 in another, and 4 in a third; how many eggs in all?
7. If a cap cost 5 shillings, a straw hat 4 shillings, and a book 1 shilling, how much do they all cost?
8. 4 birds are on the bush, 4 on the house, and 3 have flown; how many in all?
9. Eli was 4 years old 3 years ago; how old will he be 5 years hence?
10. John mixed 5 bushels of corn, 3 of rye, and 1 of oats; how many bushels in the mixture?
11. If you pay 4 dollars for a coat, 2 dollars for a hat, and 2 dollars for boots, how much do they all cost?
12. If you pay 4 cents for an orange, 3 cents for a lemon, and 2 cents for an apple, how much do the three cost?

13. Jane paid 4 cents for needles, 2 cents for thread, and had 5 cents left; how many cents had she at first?

14. William rode 5 miles with a friend, walked 3, and rode 1 mile in a stage; how far did he travel?

15. If John has eaten 3 peaches, given 5 to Mary, and 4 to Charles, how many peaches in all?

16. In a field are 4 oxen, 3 cows, and 2 calves; how many cattle in the field?

17. In a garden are 5 apple trees, 3 peach trees, and 2 plum trees; how many fruit trees in all?

18. If there are 4 boys and 5 girls present in your class, and 3 boys are absent, how many are in the class when all are present?

SECTION V.

COMBINATIONS OF 1, 2, 3, 4, 5, AND 6; THREE AT A TIME.

1. How many are 1+2+6? 1+3+6? 1+4+6?
2. How many are 2+3+6? 2+4+6? 2+5+6?
3. How many are 3+4+6? 3+5+6? 3+6+6?
4. How many are 4+4+6? 4+5+6? 4+6+6?
5. How many are 1+3+6? 1+5+6? 1+6+6?

6. William has 4 apples, John 5, and James 6; how many have they all?

7. In a stage coach were 6 men, 4 women, and 2 children; how many passengers were there?

8. Robert gathered 6 quarts of berries, James 5 quarts, and William 4 quarts; how many did they all gather?

9. In a field are 5 acres of meadow, 6 acres of

corn, and 1 of barley; how many acres are in the field?

10. 5 roses are on one bush, 2 on another, and 6 on a third; how many roses in all?

11. Seth paid 6 cents for a slate, 5 for an inkstand, and had 5 cents left; how many cents had he at first?

12. If a barrel of flour costs 6 dollars, a load of wood 2 dollars, and a cheese 3 dollars, how much do they all cost?

13. Reuben obtained 5 credit marks on Monday, 4 on Tuesday, and 6 on Wednesday; how many in all?

14. 6 pounds of sugar, 4 pounds of bread, and 2 pounds of coffee are how many pounds?

15. If a post is 3 feet in the mud, 4 feet in the water, and 6 feet above the water, how long is the post?

16. A man carried to mill 6 bushels of wheat, 5 of rye, and 1 of corn; how many bushels in all?

17. If an Algebra costs 6 shillings, an Arithmetic 3, and a Spelling Book 1 shilling, how much do they all cost?

18. 6 apples, 5 peaches, and 2 oranges, are how many pieces of fruit?

SECTION VI.

COMBINATIONS OF 1, 2, 3, 4, 5, 6, 7; THREE AT A TIME.

1. How many are 1, 2, and 7? 1, 3, and 7? 1, 4, and 7?

2. How many are $1+5+7$? $1+6+7$? $1+7+7$?

3. How many are $2+3+7$? $2+4+7$? $2+5+7$?

4. How many are 2+6+7? 3+4+7? 3+6+7?
5. How many are 3+5+7? 4+6+7? 4+5+7?

Change the orders of arrangement in each of the foregoing five lines, in every way possible; as, in example 5.

6. Add, 5+3+7; 5+7+3; 7+3+5; 7+5+3.
7. Add, 6+4+7; 6+7+4; 7+6+4; 7+4+6.
8. Add, 5+4+7; 5+7+4; 7+5+4; 7+4+5, &c.

Require the class to practice these and similar examples till they can run over them rapidly, without mistake.

SECTION VII.

COMBINATIONS EXTENDING FROM 1 TO 8; THREE AT A TIME.

1. Add, 1+2+8; 1+7+8; 1+3+8; 1+6+8.
2. Add, 1+5+8; 2+3+8; 2+7+8; 2+5+8.
3. Add, 2+6+8; 2+4+8; 3+7+8; 3+4+8.
4. Add, 3+6+8; 3+5+8; 4+7+8; 4+5+8.
5. Add, 4+6+8; 5+6+8; 5+7+8; 6+7+8.

Reverse each of the last 5, in four ways.

6. Add, 6+4+8; 6+8+4; 8+6+4; 8+4+6, &c.

SECTION VIII.

COMBINATIONS EXTENDING FROM 1 TO 9; THREE AT A TIME.

1. Add, 1+2+9; 1+4+9; 1+6+9; 1+5+9.
2. Add, 1+7+9; 1+8+9; 2+3+9; 2+5+9.

3. Add, 2+7+9; 2+4+9; 2+8+9; 2+6+9.
4. Add, 3+4+9; 3+8+9; 3+5+9; 3+7+9.
5. Add, 3+6+9; 4+5+9; 4+8+9; 4+6+9.
6. Add, 4+6+9; 5+6+9; 5+8+9; 5+7+9.
7. Add, 6+7+9; 6+8+9; 7+8+9; 7+7+9.
8. Add, 7+6+9; 7+9+6; 9+6+7; 9+7+6.

&c.

SECTION IX.

Sums of any two numbers from 10 to 19 inclusive. *Observe, that in this case the sum will contain 3 tens when the sum of the unit figures exceeds 9; otherwise the sum will contain 2 tens; as* 16+15=3 tens and 1=31, because 6+5 exceeds 9: but 14+13=2 tens and 7=27, because 4+3 is less than 10.

Add the following examples:

10 and 10	11 and 14	12 and 18	14 and 12
10 and 11	11 and 15	12 and 19	14 and 13
10 and 12	11 and 16	13 and 10	14 and 14
10 and 13	11 and 17	13 and 11	14 and 15
10 and 14	11 and 18	13 and 12	14 and 16
10 and 15	11 and 19	13 and 13	14 and 17
10 and 16	12 and 10	13 and 14	14 and 18
10 and 17	12 and 11	13 and 15	14 and 19
10 and 18	12 and 12	13 and 16	15 and 10
10 and 19	12 and 13	13 and 17	15 and 11
11 and 10	12 and 14	13 and 18	15 and 12
11 and 11	12 and 15	13 and 19	15 and 13
11 and 12	12 and 16	14 and 10	15 and 14
11 and 13	12 and 17	14 and 11	15 and 15

15 and 16	16 and 17	17 and 18	18 and 19
15 and 17	16 and 18	17 and 19	19 and 10
15 and 18	16 and 19	18 and 10	19 and 11
15 and 19	17 and 10	18 and 11	19 and 12
16 and 10	17 and 11	18 and 12	19 and 13
16 and 11	17 and 12	18 and 13	19 and 14
16 and 12	17 and 13	18 and 14	19 and 15
16 and 13	17 and 14	18 and 15	19 and 16
16 and 14	17 and 15	18 and 16	19 and 17
16 and 15	17 and 16	18 and 17	19 and 18
16 and 16	17 and 17	18 and 18	19 and 19

SECTION X.

1. How many are 70 and 60?

7 tens and 6 tens are 13 tens; 130 *Ans.*

2. How many are 150 and 170?

15 tens and 17 tens are 32 tens; 320 *Ans.*

3. How many are 18 and 14? 180 and 140?
4. How many are 15 and 13? 150 and 130?
5. How many are 11 and 17? 110 and 170?
6. How many are 80 and 90? 180 and 190?
7. How many are 16 and 19? 160 and 190?
8. How many are 17 and 18? 170 and 180?
9. How many are 28 and 37?

Solution.—Change the form of thirty seven in the mind, to seven and thirty; then 28, and 7, and 30 are how many? 28+7=35=3 tens 5, to which add 3 tens, making 6 tens 5 65. *Ans.* [Add tens to tens.]

10. How many are 78 and 15? 88, and 10?
11. How many are 48 and 34? 52, and 3 tens?

12. How many are 56 and 71? 57, and 7 tens? 12 tens 7.

13. Add 78 and 29; 8 tens 7, and 2 tens=10 tens 7=107.

14. Add 84 and 69; 93 and 6 tens=153.

15. Add 98 and 89; 107 and 80=187.

16. A farm contains 67 acres of cleared land and 24 acres of wood-land; how many acres in the farm?

17. Oneida, between Syracuse and Utica, is 26 miles from Syracuse, and 27 miles from Utica; how many miles from Syracuse to Utica?

18. If a harness costs 18 dollars and a wagon 35 dollars, how much do they both cost?

19. Paid 38 cents for a bushel of apples, and 27 cents for a bushel of potatoes; how much for both?

20. 57 bushels of corn grew on one acre of ground, and 48 bushels on another; how much on the two acres?

21. If William earns 17 dollars, Horace 15, and Richard 14 dollars in a month, how much do they all earn? 32+14=?

SECTION XI.

1. A man came from England when 23 years old, lived in New York 17 years, and has been in Ohio 32 years; how old is he? 40, and 2 and 30=?

2. If you solve 16 questions each day for 4 days, how many questions will you solve?

3. Phillip bought a pair of skates for 27 cents; he gave them and 13 cents for a sled, which he sold, and

gained 7 cents in the sale. How much did he receive for the sled?

4. A man paid 70 dollars for a carriage, 80 dollars for a horse, and 30 dollars for a harness; how much did they all cost?

5. James caught 23 fishes, William 37, and John 45; how many did they all catch?

6. What is the cost of a bushel of rye at 75 cents and a bushel of oats at 38 cents?

7. In a certain orchard, are 27 apple trees, 17 pear trees, and 18 plum trees; how many trees in the orchard?

8. A certain man was a soldier in the battle of New Orleans, 28 years old, 39 years ago: how old is he now?

9. It is 67 feet from the ground to the bell of a certain village church, and 64 feet from the bell to the top of the steeple; how high is the steeple?

10. William paid 51 cents for a book, 25 cents for a slate, and 12 cents for paper; how much did they all cost?

11. In one piece of cloth are 35 yards, in a second 37, in a third 34; how many yards in the three?

12. A certain field produces 56 bushels of wheat, 85 of corn, and 67 of oats; how many bushels in all? 56, 5 and 80, and, 7 and 60=208.

13. If you borrow 35 cents to-day, and 47 cents to-morrow, how much will you owe?

14. In January are 31 days, in February 28, and in March 31; how many days in those three months.

15. A farmer received 68 dollars for wheat, and 35 dollars for oats; how much did he receive for both?

SECTION XII.

EXAMPLES IN ADDITION, IN WHICH THE SLATE OR BLACK-BOARD MAY BE USED.

First add the units, then the tens; increasing the column of tens by adding the tens contained in the sum of the units; that is, carry one for every ten.

(1)	(2)	(3)	(4)	(5)	(6)
46	52	63	78	66	22
34	74	15	68	22	38
74	48	34	42	57	41
72	46	41	47	59	30
74	85	47	65	96	69
64	46	11	45	99	78
364					

In example (1), *begin at the bottom and add the units, pronouncing only the results*, 8, 10, 14, 18, 24, *two tens and* 4. *Write* 4, *and carry two tens to the column of tens, and we have* 8, 15, 22, 29, 32, 36; 36 *tens*=3 *hundreds and* 6 *tens. Write* 36 *on the left of the* 4 *units, and we have the sum* 364. These six examples and the following are so chosen that the units' and tens' figures in each amount are the same as in the last number in each example.

(7)	(8)	(9)	(10)	(11)	(12)
38	25	15	58	82	42
51	44	23	65	58	34
77	35	51	33	86	44
83	56	35	85	38	57
51	40	76	59	36	23
65	49	27	76	52	77

The following examples are so formed that any number found in either, is equal to the amount of all those standing above it, except that the left hand figure is thrown away. The teacher can extend this exercise indefinitely.

(13)	(14)	(15)	(16)
271	197	1112	18688
271	197	1112	18688
542	394	2224	37376
084	788	4448	74752
168	576	8896	49504
336	152	7792	99008
672	304	5584	98016
344	608	1168	96032
688	216	2336	92064
376	432	4672	84124
752	864	9344	68244
504	728	8688	36488

A series of numbers like each of the following is called an *arithmetical series:*

17. Continue 1, 4, 7, 10, to 91, by adding 3s, rapidly.
18. Continue 1, 5, 9, 13, to 101, by adding 4s.
19. Continue 1, 6, 11, 16, to 101, by adding 5s.
20. Continue 1, 7, 13, 19, to 103, by adding 6s.
21. Continue 1, 8, 15, 22, to 99, by adding 7s.
22. Continue 1, 9, 17, 25, to 97, by adding 8s.
23. Continue 1, 10, 19, 28, to 100 by adding 9s.
24. Continue 2, 5, 8, 11, to 98, by adding 3s.
25. Continue 3, 7, 11, 15, to 99, by adding 4s.
26. Continue 3, 9, 15, 21, to 99, by adding 6s.
27. Continue 4, 11, 18, 25, to 95, by adding 7s.

CHAPTER III.

SUBTRACTION.

SECTION I.

Subtraction is the process of taking one number from another; as 2 from 7 leaves 5.

The result of subtraction is the *difference* of the two numbers, or, the *remainder*.

The number from which we subtract is called the *minuend*, or *greater number*.

The number to be subtracted is called the *subtrahend*, or *less number*.

The sign of subtraction is a *short horizontal line*, (−); it is called *minus*, or *less*, as 7−5=2, seven less five is two; or, seven minus five equals two; or, seven diminished by five is two.

6−4=2.	6+3−5=4.
7−1=6.	7−1−2=4.
9−6=3.	9−6+1=4.
8−5=3.	8+2−2=8.
9−4=5.	9−2−7=0.
9−3=6.	9−2+1=8.

Subtraction is the reverse of addition; as, 2 from 5 leaves how many? is the same question as 2 and how many are 5? Answer, 3. This is seen in the following table, which is formed by changing the addition table. Commit it to memory. Read, 2 from 2, naught; 2 from 3, 1; 2 from 4, 2; &c.

SUBTRACTION TABLE.

2−2= 0	3−3= 0	4−4= 0
3−2= 1	4−3= 1	5−4= 1
4−2= 2	5−3= 2	6−4= 2
5−2= 3	6−3= 3	7−4= 3
6−2= 4	7−3= 4	8−4= 4
7−2= 5	8−3= 5	9−4= 5
8−2= 6	9−3= 6	10−4= 6
9−2= 7	10−3= 7	11−4= 7
10−2= 8	11−3= 8	12−4= 8
11−2= 9	12−3= 9	13−4= 9
12−2=10	13−3=10	14−4=10
5−5= 0	6−6= 0	7−7= 0
6−5= 1	7−6= 1	8−7= 1
7−5= 2	8−6= 2	9−7= 2
8−5= 3	9−6= 3	10−7= 3
9−5= 4	10−6= 4	11−7= 4
10−5= 5	11−6= 5	12−7= 5
11−5= 6	12−6= 6	13−7= 6
12−5= 7	13−6= 7	14−7= 7
13−5= 8	14−6= 8	15−7= 8
14−5= 9	15−6= 9	16−7= 9
15−5=10	16−6=10	17−7=10
8−8= 0	16−8= 8	13−9= 4
9−8= 1	17−8= 9	14−9= 5
10−8= 2	18−8=10	15−9= 6
11−8= 3		16−9= 7
12−8= 4	9−9= 0	17−9= 8
13−8= 5	10−9= 1	18−9= 9
14−8= 6	11−9= 2	19−9=10
15−8= 7	12−9= 3	

The difference of two *even* numbers is *even;* as, 4 from 6 leaves 2.

The difference of two *odd* numbers is *even;* as, 3 from five leaves 2.

The difference of an *even* and an *odd* number is *odd;* as, 3 from 8 leaves 5; 4 from 11 leaves 7.

When the minuend is *increased* by 10, or by any other number, the *difference* is as much *increased;* as, 5 from 7, 2; 5 from 17, 12; 5 from 27, 22: 7 from 15, 8; 7 from 25, 18; 7 from 35, 28.

To subtract 9 *from any place where the figure is less than* 9, *add* 1 *to that figure, and take* 1 *from the next left-hand place;* as, 9 from 17, 8; adding 1 to the units, and taking 1 from the tens.

Take 9 from 16; from 26; from 35.

Take 9 from 62; from 70; from 54; from 23.

To subtract 8 *from a place where the figure is less than* 8, *add* 2 *to that figure, and take* 1 *from the next left-hand place; as,* 8 from 15, 7; $=15+2-10$, adding 2, and subtracting 10.

Take 8 from 13; from 23; from 35; from 45.

To subtract 7 *when the minuend figure is less than* 7, *add* 3 *and take away* 10.

SECTION II.

1. James had 2 peaches and gave 1 to Mary; how many had he remaining? 1 from 2 leaves how many? 1 from 3? 1 from 4? 1 from 5? 1 from 6? 1 from 7? 1 from 8? 1 from 9?

2. William had 5 cents, and gave 2 for a pen; how many had he left? 2 from 5 leaves how many? 2

from 2? 2 from 3? 2 from 4? 2 from 7? 2 from 8? 2 from 9?

3. Reuben had 7 apples and gave 3 to John; how many had he left? 3 from 7 leaves how many? 3 from 4? 3 from 5? 3 from 6? 3 from 8? 3 from 9? 3 from 12?

4. John has 4 shillings, and wishes to buy a sled for 6 shillings; how many shillings does he lack? 4 from 6, how many? 4 from 5? 4 from 7? 4 from 13? 4 from 12?

5. Seth set out to go 8 miles; he rode 5 miles, and walked the remaining distance; how far did he walk? 5 from 8 leaves how many? 5 from 6? 5 from 7? 5 from 9? 5 from 14? 5 from 12? 5 from 11?

6. Edwin having 9 cents, paid 4 cents for a pencil, and 2 cents for a pen; how many cents has he remaining? (4 and 2), from 9, how many? 6 from 12? 6 from 15?

7. From a pile of wood containing 10 cords, 7 cords have been burned; how much remains? 7 from 10, how many? 7 from 12? 7 from 15? 7 from 16?

8. From 12 yards of cloth, several yards have been taken and 8 yards remain; how many yards have been taken off? 8 from 12 leaves how many? 8 from 14? 8 from 17?

9. 12 roses were on the bush, and 9 now remain; how many have been taken off? 9 from 12, how many? 9 from 15? 9 from 17? 9 from 18? 9 from 14?

10. George engaged to work 13 days; he has worked 7; how many days remain? 7 from 13 how many? 7 from 11? 7 from 16? 7 from 14?

SECTION III.

EXAMPLES IN WHICH THE SUBTRAHEND DOES NOT EXCEED 9.

1. William had 5 peaches, and has eaten 2 of them; how many remain? 5 less 2 are how many? 3 less 2? 7 less 2? 12 less 2? 21 less 2? 31 less 2? 71 less 2? 63 less 2? 42 less 2? 100 less 2?

2. Seth is 3 years younger than John, who is 7 years old; how old is Seth? 7 less 3, how many? 10 less 3? 12 less 3? 21 less 3? 34 less 3? 62 less 3?

3. Mary commenced to recite, the fourth in her class; she is now the eighth; how many places has she lost? 8 less 4, how many? 10 less 4? 12 less 4? 23 less 4? 34 less 4? 56 less 4? 90 less 4?

4. James bought 5 cents worth of paper, and paid 12 cents; how much in change should he receive? 5 from 12, how many? 5 from 14? 5 from 16? 5 from 24? 5 from 33? 5 from 42? 5 from 51?

5. Henry having 18 cents, paid 6 cents for a book; how much should he receive in change? 6 from 18, how many? 6 from 13? 6 from 12? 6 from 25? 6 from 24? 6 from 43? 6 from 52? 6 from 71? 6 from 80?

6. Charles having 25 cents, paid 7 cents for a knife; how many cents remain? 7 from 25 how many? 7 from 15? 7 from 34? 7 from 56? 7 from 68? 7 from 83?

7. Reuben bought a kite for 37 cents, and sold it for 8 cents less; how much did he receive for it? 37 less 8, how many? 45 less 8? 17 less 8? 67 less 8? 63 less 8? 91 less 8? 100 less 8?

8. From a piece of cloth containing 32 yards, William bought 5 yards, and James 4; how many yards remain? 9 from 32, how many? 9 from 17? 9 from 47? 9 from 28? 9 from 75? 9 from 46? 9 from 53?

SECTION IV.

As addition teaches to count forward, adding 1 or more at a time, so substraction teaches to count backward, taking away 1 or more at a time, as in the following examples. Each number having the sign (—) before it, is to be subtracted successively?

1. What is each remainder in 18—5—6—3—4? *Answer*, 13, 7, 4, 0.
2. Find all the remainders in 18—4—3—6—5. *Answer*, 14, 11, 5, 0.
3. Find the remainders in 35—6—4—3—2—5. *Ans.* last rem. 15.
4. How many is 47—6—5—7—8—14 *Ans.* 20.
5. How many is 63—9—9—8—8? *Ans.* 29.
6. How many is 36—4—5—8—3?
7. How many is 85—2—7—6—5?
8. How many is 73—4—5—8—3?
9. How many is 54—7—3—6—4?
10. How many is 33—9—6—4—1?
11. How many is 95—8—9—7—6?
12. A man having 45 dollars, paid 6 dollars for a barrel of flour, 5 dollars for fuel, and 7 dollars for a hundred weight of beef; how many dollars had he left?
13. Jane having 50 cents, paid 8 cents for tape,

9 cents for pins, 6 cents for needles, and 5 cents for thread; how many cents had she left?

14. Andrew having 56 cents, gave 5 cents to one beggar, 3 cents to a second, 4 to a third, and 6 to a fourth; how many cents had he left?

15. A man died 56 years old, having come from England, and lived in New York 6 years, in Ohio 4 years, and in Indiana 8 years; at what age did he come from England?

SECTION V.

TO SUBTRACT ANY NUMBER BETWEEN 10 AND 100.

First, subtract the units; then subtract the tens from the tens in this remainder.

1. A man having 85 sheep, sold 47; how many remain? 85 less (7 and 40,)=78 less 40=38. *Ans.*

2. A man having 73 dollars, paid a debt of 28 dollars; how many dollars had he left? 73—8—20=65—20.

3. James having 75 cents, bought a book for 39 cents; how many cents has he remaining? 75—9—30.

4. In a lesson containing 45 questions, 16 were answered incorrectly; how many were answered correctly?

5. A merchant paid 95 dollars for goods, which became damaged, and he sold them for 68 dollars; how much did he lose?

6. In July and August, 62 days, were 23 cloudy days; how many were fair days?

7. In a hot day, the thermometer told 91 degrees; how many degrees was that above the freezing point, which is 32 degrees?

8. London is in 51 degrees north latitude, and Philadelphia 40 degrees. How many degrees is London farther north than Philadelphia?

9. How many is 65—47? 34—18?

10. How many is 36—14? 43—25?

11. How many is 85—47? 63—28?

12. How many is 74—52? 82—65?

13. How many is 93—64? 93—75?

14. How many is 85—23? 85—46? 85—39? 85—27? 85—63? 85—73? 85—79?

15. From 63 take 27, 36, 50, 48, 53.

16. From 91 take 12, 15, 17, 25, 38.

17. From 100 take 13, 16, 23, 27, 48.

18. From 120 take 40, 30, 50, 60, 90.

19. From 123 take 23, 43, 73, 93, 53.

20. From 256 take 16, 26, 44, 64, 74.

21. From 328 take 18, 28, 38, 49, 59.

22. From 409 take 19, 39, 59, 67, 87.

23. From 523 take 51, 61, 91, 83, 93.

24. From 685 take 87, 97, 55, 65, 75.

SECTION VI.

SUBTRACTION OF NUMBERS GREATER THAN 100, TO BE PERFORMED ON THE SLATE.

1. Suppose you have a treasury containing 645 dollars, consisting of 6 one hundred dollar bills, 5 ten dollar bills, and 4 one dollar bills, and you take from it 321 dollars; how many dollars will be left?

OPERATION.

Minuend, 645
Subtrahend, 321
Remainder, 324

Beginning at the right hand, take 1 from 5, 4; 2 (tens) from 4 (tens), 2 (tens); 3 (hundreds) from 6 (hundreds), 3 (hundreds). *Answer*, 324 dollars.

2. Suppose you have a treasury containing 523 dollars, consisting of 5 one hundred dollar bills, 2 ten dollar bills, and 3 one dollar bills; and you wish to take from it 148 dollars; how many dollars will be left?

OPERATION.

Minuend, 523
Subtrahend, 148
Remainder, 375

Beginning with the units, you would take 8 from 3, but you cannot. Add 10 units (one dollar bills) to the 3, making 13; then, 8 from 13 leaves 5, written under the 8. Now take away 5 tens, one more than the 4 in the subtrahend, to pay for the 10 ones which you added to the 3. But 5 from 2 you cannot: add 10 tens to the 2 tens, making 12 tens. 5 from 12, 7 (tens), written under the 4 tens. Now take away 2 hundreds, one more than the 1 in the subtrahend, to pay for the 10 tens added to the 2, 2 from 5, 3 (hundreds), written under the 1 hundred.

Therefore, for subtraction we have the following

RULE.

I.—*Place the subtrahend under the minuend, so that units of the same order shall stand in the same column.*

II.—*Commencing at the right hand, take each figure from the one directly over it, and set down the remainder.*

III.—*But if the upper figure be the less, add ten to it, subtract from the figure so increased, and carry one to the next figure in the subtrahend.*

Proof.—1. Add the remainder and subtrahend; the sum will equal the minuend.

2. If the remainder be taken from the minuend, it will leave the subtrahend.

(3)	(4)
From 83251	From 8375248
Take 12061	Take 7484569
Rem. 71190	Rem. 890679
(5)	(6)
From 6387259	From 28460302
Take 1278061	Take 19046859

(7)	(8)	(9)
From 682745 pounds	890004 yards	560048 lb
Take 328478 pounds.	730005 yards.	123489 lb.

10. A man having 4036 dollars, bought a house for 2375 dollars; how many dollars has he remaining? *Ans.* 1661 dollars.

11. If I should borrow of my neighbor 1000 dollars, and 30 days afterwards should pay him 475 dollars, what should I still owe him? *Ans.* 525 dollars.

12. A drover bought cattle to the amount of 2500 dollars, and in the sale of them lost 359 dollars; how much did he receive for them?

13. The same drover bought another lot of cattle for 2650 dollars, and in the sale of them lost 125 dollars; how much did he receive for both lots?

Ans. 4666 dollars.

14. The distance from Albany to Buffalo is 320 miles by railroad; from Albany to Rochester is 252 miles; how far is it from Rochester to Buffalo?

15. America was discovered by Columbus in 1492; how many years since?

16. George Washington was born in 1732, and died in 1799; how old was he?

17. In 1840, the state of New York contained 2428921 inhabitants; in 1850, 3097394; what was the increase of population in this state in the intervening ten years? *Ans.* 668473.

18. The population of the United States was, in 1840, 17063353; in 1850, 23263488; what was the increase of population in the United States in those ten years? *Ans.* 6200135.

SECTION VII.

FEDERAL MONEY.

Federal Money is the currency of the United States, in which

10 mills, marked (*m*), make	1 cent,	*c.*
10 cents	1 dime,	*d.*
10 dimes	1 dollar,	$.
10 dollars	1 eagle,	*E.*

The *dollar* is the *unit*, and is distinguished in notation by placing this character ($) before it; and a point, called a *decimal point*, or *separatrix*, is placed on the left of dimes, separating them from dollars; as, $25.18, read 25 dollars 18 cents; $1.853, 1 dollar 85 cents 3 mills; $0.25 is 25 cents; $0.032 is 3 cents 2 mills; that is, the first figure on the left of the separatrix is dollars, next tens of dollars, &c.; the

first on the right of the point is dimes, or tens o cents, next cents, next mills. Thus:

NUMERATION TABLE.

Hundreds of dollars.	Tens of dollars.	Dollars.	Dimes.	Cents.	Mills.	
			0	0	5	= 5 mills.
			0	5	8	= 5 cents 8 mills, or 58 mills.
			6	2	5	= 62*c.* 5*m.*, or 625 mills.
	8	5	0	4	2	= $85 4*c.* 2*m.* = 85042 mills.
1	2	6	4	3	7	= $126 43*c.* 7*m.* = 126437 mills.

We may change the unit in Federal money, by re moving the decimal point, and thus reduce the sum from one name to another, without changing its value; as $1.252=1252 mills; $12.04=1204 cents $2.60=26 dimes, &c.; $150=15,000 cents=15000(mills; so, also, 5 mills=$.005; 2 cents=$.02 that is,

To change the unit from dollars to cents, remov the unit's place two places to the right hand; to mills three places.

In the notation of Federal money, as in whole num bers, each intermediate place, from the highest to the lowest, must be occupied by a digit or a cipher; as 25 dollars and 2 cents, must be written $25.02.

1. How many cents are there in each of the follow ing sums? $25; $250; $[illegible]50; $675; $895 $878; $401; $505; $6758; $7006.

2. How many mills in $1? $2? $25? $275? $2.50? $38.54? $27.95? $3.755? $4.262? $1.125.

3. How many dollars in 100 cents? 278 cents? 1400c.? 1238c.? 67858c.? 78251c? 868420c.?

4. How many dimes in 10c.? 20c.? 28c.? 3695c.? 4824c.? 37850c.?

SECTION VIII.

TO ADD OR SUBTRACT FEDERAL MONEY.

Place dollars under dollars, cents under cents, &c.; add or subtract as in whole numbers, and place the separatrix in the result, in the same column with the separating points above.

EXAMPLES FOR ADDITION.

(1)	(2)	(3)	(4)
$125.367	$328.042	$315.20	$12.13
152.30	63.20	278.55	6.14
278.005	582	114.63	1.18
326.05	051	17.12	2.19
$881,722	391,875		

5. What sum must be paid for a coat at $5.875, a vest at $3.19, and a pair of shoes at $2.625?

Ans. $11.69.

6. A farmer sold produce as follows; wheat for $300: corn for 98 dollars 94 cents; hay for 55 dollars 12 cents; and oats for 18 dollars 6 cents; what was his amount of sales? *Ans.* $472.12.

7. If a quarter of beef cost, 7 dollars 8 cents; a barrel of flour $5.56; a barrel of sugar $15; and a

bag of coffee 10 dollars 5 cents, what is the cost of the whole? *Ans.* $37.69.

8. Bought 6 pounds of coffee for 1 dollar 12½ cents, (1 dollar 12 cents 5 mills, $1.125); butter for 2 dollars 9 cents; a looking glass at $3.56; a table at 5 dollars; and a set of chairs at $9.50; what is the amount? *Ans.* $21.275.

SECTION IX.

EXAMPLES FOR SUBTRACTION.

	(1)	(2)	(3)	(4)
From	$183.252	327.18	125.16	18.75
Take	84.267	124.16	64.12	9.80
Rem.	$98.985			

5. A man borrowed 56 dollars 4 cents, and paid 40 dollars 9 cents; how much does he still owe?

6. A man owed $105.28, and paid all but 6 dollars 30 cents; how much did he pay? *Ans.* $98.98.

7. A grocer paid $65.28 for two hogsheads of molasses, and sold them for 79 dollars 8 cents; how much did he gain by the bargain? *Ans.* $9.80.

8. A merchant bought a quantity of silks for $245,38, which becoming damaged he sold for 150 dollars 6 cents; how much did he lose?

Ans. $95.32.

9. From 25 dollars take 25 cents; from 16 dollars take 16 mills; from 27 dimes take 14 cents; what is the sum of the three remainders? *Ans.* $43.294.

10. From 16 dollars take 5 mills.

11. From 100 dollars take 10 cents 5 mills.

12. From 1000 dollars take 1000 mills.

SECTION X.

QUESTIONS SOLVED BY ADDITION AND SUBTRACTION. MENTAL EXERCISES.

1. A boy having 28 cents, gave 12 cents for a knife, and 9 cents for a ball; how many cents had he left?

2. From a piece of gingham containing 28 yards, Mary purchased 8 yards, and Jane 7 yards; how much was left in the piece?

3. A market boy having 27 quarts of cherries, William bought 6 quarts of them, James 5 quarts, John 7 quarts, and a baker the rest; how many quarts did the baker buy?

4. Julia having 30 cents, purchased a slate for 9 cents, a sponge for 3 cents, pencils for 4 cents, and writing paper for 12 cents; how many cents had she left? 9, 3, 4, 12, and how many make 30? *Ans.* 2.

5. A boy sold two dozen eggs at 14 cents a dozen, and 4 quarts of strawberries at 8 cents a quart, and received a book worth 50 cents; how many cents in money are due the merchant or the boy? 14+14+8+8+8+8−50=how many?

Ans. Due the boy 10 cents.

6. A Jeweller took a watch at 15 dollars, in part payment for one at 37 dollars; how much money will pay the balance?

7. From a flock of 97 sheep, 25 were sold at one time, and 16 at another; how many sheep remain in the flock? 97−41=how many?

8. A dairyman sold a firkin of butter for 12 dollars, two cheeses for 8 dollars, and received a barrel of flour worth 6 dollars; how much is the dairyman's due?

9. Jane bought cambric for 12 cents, thread for 7 cents, and a paper of needles for 6 cents; she handed the merchant 50 cents. How much change is her due?

10. A man set out on a journey of 67 miles; the first day he went 25 miles, the second day 31 miles; how many miles must he go to finish his journey?

11. A merchant bought a hogshead of molasses for 35 dollars, and paid 5 dollars for freight; for how much must he sell it to gain 12 dollars?

12. A man bought a yoke of oxen for 85 dollars; their labor was worth 37 dollars, and their keeping cost 12 dollars,—he then sold them for 74 dollars; how much did he gain or lose?

13. A jeweller bought a watch for 25 dollars, a chain for 12 dollars, and a key for 8 dollars, and sold them for 50 dollars; how much did he gain in the sale?

14. Julia bought a comb for 31 cents, some pins for 9 cents, tape for 8 cents, needles for 6 cents, and thread for 5 cents; she gave 1 dollar; how much change is her due?

15. A drover bought 10 sheep of one man, of another 8, of a third 7, of a fourth 9; to a fourth he sold 6, to a fifth he sold 11; how many had he then?
$10+8+7+9-6-11=$how many?

16. Find the value of $16+8-9+7$.

17. Find the value of $12+8+12-28$.

18. How many is $45+17-39$?

19. How many is $165-25-30$?

20. How many is $214-18-16$?

21. How many is $918-28-19$?

22. How many is $75+7+8+3+9+4-12$?

23. How many is $83+6-5+8-7+2-1$?

24. How many is $56+10-9+8-6-3-4$?

SECTION XI.

ADDITION AND SUBTRACTION.

1. 8+7+what number=21?
2. 3+6+7+what number=23?
3. What number+5−6+4=12?
4. What number+2−7+1=4?
5. What number+5−6−3=0?
6. What number−7−2−1=0?
7. What number+10−10=0?
8. What number+2−10−12=0?
9. 8+6+3−what number=12?
10. 6+7−3−what number=10?
11. 5+3+4−what number=7?
12. 7+2+3−what number=5?

13. John has 12 cents; Peter has 19 cents; James has 4 cents more than John and Peter; Joseph has 15 cents less than James: how many cents has Joseph? how many have they all?

12+19+35+20=how many?

14. Four boys bought a melon for 25 cents; A paid 5 cents; B, 6 cents; C, 4 cents; and D, the rest: how many cents paid D?

15. James having 75 cents, paid 18 cents for a book, 12 cents for paper, and 15 cents for a bottle of ink; how many cents had he left?

16. A man sold a horse for 65 dollars, and a harness for 21 dollars, and in the sale gained 12 dollars; how much did they cost him?

17. A merchant had two casks of wine, one containing 42 gallons, and the other 38 gallons. Having sold 29 gallons from the first, and 17

gallons from the second, how much remains unsold? *Ans.* 80—46=34 gallons.

18. When in June the day is 15 hours long, how long is the night? A day and night=24 hours.

19. When in December the day is 9 hours long, how long is the night?

20. Thomas having 81 cents, paid 31 cents for a knife, and 37 cents for a pair of skates, how many cents had he left?

21. A man set out to go 87 miles; he went 32 miles the first day, and 29 miles the second; how far must he go on the third day to finish his journey?

SECTION XII.

EXERCISES FOR THE SLATE.

1. A farmer, having 827 sheep, sold to A 115 of them; to B, 96; to C, 275; and to D, 120: how many had he remaining? *Ans.* 221 sheep.

2. A market woman, having 150 oranges, sold to one person 12 of them; to another, 35; to a third, 78; to a fourth, 17; and to a fifth, 8: how many had she remaining? *Ans.* None.

3. There is a farm containing 740 acres, of which 25 acres bear corn; 175, rye; 167, oats; 90, wheat; 185 acres are pasture, and the rest meadow: how many acres are meadow? *Ans.* 98 acres.

4. A man, owing $8750, paid at one time $475.25; at another, $654.80; at another, $753: how much did he still owe? *Ans.* $6866.95.

5. From a county containing 30578 inhabitants,

two towns were set off to an adjoining county; one of those towns contained 1475, and the other 1350: how many inhabitants were left in the county?

Ans. 27758.

6. How many times can 875 be taken from 3003? and what is the last remainder?

Ans. 8 times, and 378 remain.

7. A has $35.25; B, 26.45; C has as much money as both A and B, wanting 15 dollars: how much has C? *Ans.* $46.70.

8. From New York to Dunkirk, by the Erie railroad, is 469 miles; from New York to Binghamton, 225 miles; from Binghamton to Elmira, 58 miles: what is the distance from Elmira to Dunkirk?

Ans. 186 miles.

9. If you earn $5.75 a week for 4 weeks, how much more must be earned to make thirty dollars?

10. What number is that which, if you subtract it from 1275, will leave a remainder of 824. *Ans.* 451.

11. What number added to 628 and 923 will make the sum 2000? *Ans.* 449.

12. A has $12.75; B has $3.99 less than A; and C lacks 2 cents of having as much money as A and B: how much has C? *Ans.* $21.49.

13. A man paid out $17.87, then he received $44.62, and found that he had $50 in his possession; how much money had he at first? *Ans.* $23.25.

14. James has such a sum that if you add to it $16.44, and take from it $28.44, he will have $8 left; how much money has he? *Ans.* $20.

15. What number diminished by 245, and by 148, there will remain 294? *Ans.* 687.

CHAPTER IV.

MULTIPLICATION.

SECTION I.

1. What cost 3 apples at 2 cents a piece?

Solution. 3 apples will cost 3 times as much as 1 apple. If 1 apple costs 2 cents, 3 apples cost 3 times 2 cents, which is 6 cents. *Ans.*

The pupil should give the reasoning process on every question, until he is quite familiar with it.

2. What cost 4 lemons at 3 cents a piece.

3. At 5 cents a pound for rice, what cost 2 pounds? 4 pounds? 3 pounds?

4. At 6 cents a pound for pork, what cost 3 pounds? 2 pounds? 4 pounds? 5 pounds?

5. At 7 cents a pound for sugar, what cost 2 pounds? 4 pounds? 3 pounds? 5 pounds?

6. At 8 dollars a barrel for flour, what cost 2 barrels? 3 barrels 4 barrels? 5 barrels?

7. If you can walk 4 miles in 1 hour, how far can you walk in 3 hours? 6 hours? 4 hours?

The process of repeating one number as many times as there are units in another number, is called *multiplication.*

The number to be multiplied is the *multiplicand.*

The number to multiply by is the *multiplier.*

The result of multiplication is the *product.*

The multiplicand and multiplier together are called

the *factors* of the product; as, 4 and 3 are factors of 12.

The oblique cross (×), denotes multiplication; as 4×3; 4 multiplied by 3, or 4 times 3.

MULTIPLICATION TABLE.

2 times 0 = 0	3× 0= 0	4× 0= 0
2× 1= 2	3× 1= 3	4× 1= 4
2× 2= 4	3× 2= 6	4× 2= 8
2× 3= 6	3× 3= 9	4× 3=12
2× 4= 8	3× 4=12	4× 4=16
2× 5=10	3× 5=15	4× 5=20
2× 6=12	3× 6=18	4× 6=24
2× 7=14	3× 7=21	4× 7=28
2× 8=16	3× 8=24	4× 8=32
2× 9=18	3× 9=27	4× 9=36
2×10=20	3×10=30	4×10=40
2×11=22	3×12=33	4×11=44
2×12=24	3×12=36	4×12=48

5× 0= 0	6× 0= 0	7× 0= 0
5× 1= 5	6× 1= 6	7× 1= 7
5× 2=10	6× 2=12	7× 2=14
5× 3=15	6× 3=18	7× 3=21
5× 4=20	6× 4=24	7× 4=28
5× 5=25	6× 5=30	7× 5=35
5× 6=30	6× 6=36	7× 6=42
5× 7=35	6× 7=42	7× 7=49
5× 8=40	6× 8=48	7× 8=56
5× 9=45	6× 9=54	7× 9=63
5×10=50	6×10=60	7×10=70
5×11=55	6×11=66	7×11=77
5×12=60	6×12=72	7×12=84

8× 0= 0	9× 0= 0	10× 0= 0
8× 1= 8	9× 1= 9	10× 1= 10
8× 2=16	9× 2= 18	10× 2= 20
8× 3=24	9× 3= 27	10× 3= 30
8× 4=32	9× 4= 36	10× 4= 40
8× 5=40	9× 5= 45	10× 5= 50
8× 6=48	9× 6= 54	10× 6= 60
8× 7=56	9× 7= 63	10× 7= 70
8× 8=64	9× 8= 72	10× 8= 80
8× 9=72	9× 9= 81	10× 9= 90
8×10=80	9×10= 90	10×10=100
8×11=88	9×11= 99	10×11=110
8×12=96	9×12=108	10×12=120

11× 0= 0	12× 0= 0
11× 1= 11	12× 1= 12
11× 2= 22	12× 2= 24
11× 3= 33	12× 3= 36
11× 4= 44	12× 4= 48
11× 5= 55	12× 5= 60
11× 6= 66	12× 6= 72
11× 7= 77	12× 7= 84
11× 8= 88	12× 8= 96
11× 9= 99	12× 9=108
11×10=110	12×10=120
11×11=121	12×11=132
11×12=132	12×12=144

Any product of two or more whole numbers is called a *composite number;* as, $15=5\times3$.

A composite number may be composed of 3 or more factors; as, $30=2\times3\times5$.

A number which is not composed of other whole numbers as factors, (besides itself and one,) is a *prime number;* as, 2, 3, 5, 7, 11, are prime numbers.

The multiplying of a number by a whole number may be effected by addition; as, $4 \times 3 = 4+4+4$; setting down the multiplicand as many times as there are units in the multiplier, and adding; but to multiply by a fraction requires division; as $\frac{2}{3}$ of 12 is 8.

The multiplication table shows the following properties of numbers.

1. The product of two *even* numbers is *even*.
2. The product of an *even* and an *odd* number is *even*.
3. The product of two *odd* numbers is *odd*.
4. Every product of 5, ends with 0, or 5.
5. Every product of 9, not exceeding 10 times 9, makes a number having the tens figure one less than the number of 9s, and having the sum of its figures equal to 9.

Every product of 10, ends with 0.

Every product of 11, not exceeding 9 times 11, consits of 2 figures alike, the same as the number of 11s.

SECTION II.

Let the pupil be required to pronounce, at sight, the product of each number by the number standing under it, at the head of each of the four following sections. Let the class in concert practice these exercises until they can go through them readily. Thus, in the first example pronouncing—Sixteen, four, twenty-four, ten, eighteen, &c.

1. Multiply	8	2	12	5	9	7	3	11	6	10	4
by	2	2	2	2	2	2	2	2	2	2	2

2. Multiply	8	2	12	5	9	7	3	11	6	10	4
by	3	3	3	3	3	3	3	3	3	3	3
3. Multiply	8	2	12	5	9	7	3	11	6	10	4
by	4	4	4	4	4	4	4	4	4	4	4

What cost 4 lead pencils, at 5 cents apiece?

Solution.—4 pencils will cost 4 times as much as 1 pencil. If 1 pencil cost 5 cents, 4 pencils will cost 4 times 5 cents, 20 cents. *Ans.*

3. At 6 cents a quart for cherries, what cost 2 quarts? 4 quarts? 3 quarts?

4. At 4 cents apiece for oranges, what cost 4 oranges? 3 oranges? 2 oranges?

5. At 7 cents a yard for ribbon, what cost 3 yards? 4 yards? 2 yards?

6. If a boat moves 8 miles an hour, how far will it move in 4 hours? 2 hours? 3 hours?

7. If you set trees, 9 trees in each row, how many trees in 4 rows? 3 rows? 2 rows?

8. Ten cents make a dime; how many cents in 4 dimes? 3 dimes? 2 dimes?

9. If a steamboat goes 11 miles an hour, how far will it go in 3 hours? 2 hours? 4 hours?

10. At 12 cents a dozen for eggs, what cost 4 dozen? 3 dozen? 2 dozen?

11. A man bought 4 pounds of beef at 8 cents a pound, and paid 37 cents; how much change is his due?

12. Mary bought 3 yards of cambric at 12 cents a yard, and paid 50 cents; how much change is her due?

13. Four chandeliers have 8 lamps each, and all the lamps but 4 are lighted; how many lamps are lighted?

SECTION III.

1. Multiply	4	10	6	11	3	7	9	5	12	2	8
by	5	5	5	5	5	5	5	5	5	5	5
2. Multiply	4	10	6	11	3	7	9	5	12	2	8
by	6	6	6	6	6	6	6	6	6	6	6
3. Multiply	4	10	6	11	3	7	9	5	12	2	8
by	7	7	7	7	7	7	7	7	7	7	7

4. What cost 5 pencils at 6 cents apiece? 6 pencils at 5 cents apiece?

5. The railroad fare from Albany to Boston being 5 dollars for 1 passenger, how much will 2 passengers pay? 3 passengers? 4 passengers? 8? 6? 7? 5? 12? 10? 11? 9 passengers?

6. At 6 cents a pound for nails, what cost 4 pounds? 2 pounds? 3 pounds? 8? 6? 7? 5? 12? 10? 9? 11 pounds?

7. At 7 dollars a barrel for flour, what cost 2 barrels? 4 barrels? 3 barrels? 8? 6? 7? 5? 12? 10? 11? 9 barrels?

8. What cost 6 pounds of cheese at 8 cents a pound, and 4 pounds of beef at 7 cents a pound?

Ans. 48+28=76 cents.

9. Henry went to a store with 50 cents, and bought 4 inkstands at 6 cents apiece, and 6 skeins of silk at 3 cents apiece; how many cents had he in return?

10. A steamboat going 12 miles an hour overtakes a sloop sailing 5 miles an hour; how far ahead of the sloop will the steamboat be in 1 hour? in 2 hours? 4 hours? 8 hours? 7 hours? 9 hours? 11 hours? 12 hours?

11. Two canal boats, each moving 3 miles an hour, meet and pass each other; how far apart will they be in 1 hour? in 2 hours? 3 hours? 5 hours? 6 hours? 8 hours? 10 hours? 11 hours? 12 hours?

12. If you buy 3 quarts of milk at 4 cents a quart, and 4 pounds of bread at 3 cents a pound, and pay 25 cents, how much change will you receive?

13. 3 times 6 and 2 times 9 are how many?

14. $3\times4+2\times6$=how many? *Ans.* 24.

15. $3\times8+4\times6$=how many?

16. $4\times8-4\times5$=how many?

17. $4\times(8-5)$=how many? The parenthesis shows that the numbers $(8-5)$ are to be made a simple number before multiplication. $(8-5)=3$, $3\times4=12$. *Ans.*

18. $3\times9-3\times7$=how many? *Ans.* 6.
$3\times(9-7)$=how many? *Ans.* 6.

19. $4\times5+4\times3$=how many?
$4\times(5+3)$=how many?

20. $6\times7-35$=how many?

21. $42-5\times7$=how many?

22. 6 times 4 is how many times 3?

23. 8 times 6 is how many times 8?

24. 4 times 5 is how many times 2?

SECTION IV.

1. Multiply	4	7	12	5	8	2	11	9	6	10	3
by	8	8	8	8	8	8	8	8	8	8	8
2. Multiply	4	7	12	5	8	2	11	9	6	10	3
by	9	9	9	9	9	9	9	9	9	9	9

3. Multiply 4 7 12 5 8 2 11 9 6 10 3
by 10 10 10 10 10 10 10 10 10 10 10

4. At 8 cents a pound for sugar, what cost 2 pounds? 4 pounds? 3 pounds? 8? 6? 7? 5? 12? 10? 11? 9 pounds?

5. At 9 cents a yard for cotton cloth, what cost 4 yards? 2 yards? 8 yards? 8? 6? 7? 5? 12? 10? 9? 11 yards?

6. At 10 cents a pound for coffee, what cost 2 pounds? 3 pounds? 4? 8? 7? 10? 11? 12 pounds?

7. 4 times 8 is how many times 4?

8. If you study 6 hours each day, how many hours will you study in 2 days? 3 days? 4 days? 5 days? in 2 weeks, devoting 5 days each week to study?

9. If you read 8 pages of history each day, how many pages will you read in 2 days? 3 days? 4 days? 12 days?

10. James bought 4 pieces of crockery at 7 cents apiece, and 5 pieces at 8 cents apiece; and paid 75 cents: how much change is his due?

$75-(4\times7+5\times8)=$how many cents? *Ans.* 7 cts.

11. Helen bought 7 yards of calico at 12 cents a yard, and 4 spools of thread at 4 cents each; and paid 1 dollar: how much change is her due?

12. Julia bought 7 skeins of silk at 3 cents a skein, and 4 sticks of twist at 2 cents a stick; and paid 31 cents: how much change is her due?

13. In 4 stages, each carrying 9 passengers; and 3 stages, each carrying 12 passengers: how many passengers in all?

14. Eight quarts make 1 peck; 4 pecks make 1 bushel: how many quarts in 1 bushel? in 2 bushels?

15. Four farthings make 1 penny; 12 pence make 1 shilling: how many farthings in 1 shilling? in 2 shillings?

16. Ten cents make 1 dime; how many cents in 2 dimes? in 3 dimes? 4 dimes? 9 dimes? 10 dimes?

17. Ten dimes make 1 dollar; how many dimes in 2 dollars? in 3 dollars? 5 dollars? 7 dollars? 10 dollars?

18. A man, having 1 dollar, bought 9 pounds of beef at 9 cents a pound; how many cents had he left?

19. James bought 5 yards of shirting at 10 cents a yard, and paid 6 quarts of strawberries at 8 cents a quart: how much change must he pay, or receive?

Ans. Pay 2 cents.

20. A man bought 9 pounds of veal at 6 cents a pound, and gave 75 cents; how much change is his due?

21. A man, owing 75 dollars, paid 7 tons of hay at 9 dollars a ton; how much remains due?

22. 8 times 9—6 times 9=how many times 9?

23. 7 times 12—6 times 12=how many times 12?

24. 5 times 8—4 times 8=how many times 8?

SECTION V.

1. Multiply	3	10	6	9	11	2	8	5	12	7	4
by	11	11	11	11	11	11	11	11	11	11	11
2. Multiply	3	10	6	9	11	2	8	5	12	7	4
by	12	12	12	12	12	12	12	12	12	12	12

3. When hay is worth 11 dollars a ton, what cost 2 tons? 4 tons? 3 tons? 8 tons? 6 tons? 7 tons? 5 tons? 12 tons? 10 tons? 11 tons? 9 tons?

4. At 12 cents a piece for candlesticks, what will 2 candlesticks cost? 3? 4? 8? 6? 7? 5? 12? 10? 11? 9?

5. Paying 12 cents a pound for butter, what will 2 pounds cost? 4 pounds? 3? 8? 6? 7? 5? 12? 10? 9? 11?

6. If 11 pounds of butter are given for 1 yard of cloth, how many pounds of butter will pay for 3 yards? 4 yards? 8 yards? 10 yards? 11 yards? 12 yards?

7. A man owing 100 dollars, paid 7 tons of hay at 12 dollars a ton; how much remains due?

8. 12 is how many times 2? Twice 12 is how many times 2? 3 times 12 is how many times 2? 8 times 12 is how many times 2?

Since 12 is 6 times 2, any number of 12s, is 6 times as many 2s. Therefore two 12s=6 times two 2s=twelve 2s, and eight 12s=six times eight 2s, or 48 times 2.

9. 8 times 4, is how many times 2?

10. 8 times 9 is how many times 3?

11. 12 times 9 is how many times 3? how many times 6?

12. A lady bought 6 yards of silk at 2 dollars a yard, 2 shawls at 8 dollars each, and 2 bonnets at 4 dollars each; she paid 8 five dollar bills: how much should she receive in return?

13. John has 8 dollars; William has 3 times as much money lacking 3 dollars; how many dollars have they both.

14. John has 8 dollars, William has 3 times as

much money lacking 3 dollars, and Seth has as much lacking 5 dollars, as John and William; how much money have the three? *Ans.* 53 dollars.

15. 3 times 12, and twice 11 make what number?

SECTION VI.

TO MULTIPLY A LARGE NUMBER BY ANY NUMBER NOT EXCEEDING 12, ON THE SLATE.

Observe that—

2 ×3= 6; units into units are units.
20 ×3= 60; units into tens are tens.
200×3=600; units into hundreds, are hundreds, &c.

1. Multiply 1234, by 7. The product will be equal to 7 times 4+7 times 30+7 times 200+7 times 1000.

Operation		Or thus
1234		1234
7		7
28	=7 times 4	8638. *Ans.*
21	=7 times 3 tens	
14	=7 times 2 hundred	
7	=7 times 1 thousand	
Sum 8638	*Ans.*	

7 times 4 is 28. 7 times 3 is 21, and 2 to carry makes 23. 7 times 2 is 14, and 2 to carry makes 16. 7 times 1 is 7, and 1 to carry makes 8.

Begin at the right hand, and multiply, carrying one for every ten, as in simple addition.

EXAMPLES.

	(2)	(3)	(4)	(5)
Multiply	5275	7852	3105	42875
by	6	9	4	8
Product	31650	70668	12420	343000

6. $785 \times 3 = 2355$
7. $655 \times 4 = 2620$
8. $708 \times 5 = 3540$
9. $609 \times 9 = 5481$
10. $827 \times 11 = 9097$
11. $752 \times 12 = 9024$
12. 7298×5
13. 6405×6
14. 7899×8
15. 6004×9
16. 7029×7
17. 6847×3

18. What will 3 acres of land cost at 125 dollars an acre. *Ans.* 375 dollars.

19. What will 7 horses cost at $115 a head?

20. What cost 873 barrels of flour at $7 a barrel? 873 times $7, is the same as 7 times $873. For if the flour was $1 a barrel, 873 barrels would cost $873; but, being $7 a barrel, the cost will be 7 times as much, $873 \times 7 = $6111. *Ans.*

21. What cost 834 barrels of pork at $12 a barrel? *Ans.* $10008.

22. What cost 728 yards of cloth at $3 a yard?

23. What cost 3 yards of cloth at $7.28 a yard? *Ans.* $21.84.

Multiply dollars and cents like whole numbers, being careful to give two places to the cents.

24. What cost 9 hats at 3 dollars 6 cents a piece? *Ans.* $27.54.

25. What cost 7 horses, at $125.84 cents each? *Ans.* $880.88.

26. If 6 persons have $875.37 each, how much have they all? *Ans.* $5252.22.

SECTION VII.

TO UNDERSTAND THE RESULT OF MULTIPLYING TOGETHER ANY TWO FIGURES, WHETHER UNITS, TENS, HUNDREDS, &c.

Observe that—

2 × 3 = 6; units into units are units.
2 × 30 = 60; units into tens are tens.
20 × 30 = 600; tens into tens are hundreds.
20 × 300 = 6000; tens into hundreds are thous'ds.
200 × 300 = 60000; hunds. into hunds. are 10 thou's.

That is, whatever are the distances of the two figures multiplied, from the units' place, the distance of the product from the units' place will be equal to the sum of the two distances; as in the following example:

Multiply 1234 by 567.

	100 thou's.	10 thou's.	Thou's.	Hund's.	Tens.	Units.
Multiplicand .			1	2	3	4
Multiplier . .				5	6	7
Prod. by 7 . .			7	14	21	28
" " 6 . .		6	12	18	24	
" " 5 . .	5	10	15	20		
Whole product	5	16	34	52	45	28

Now set down and add 28 units.

45 tens.
52 hundreds.
34 thousands.
16 ten thousands.
5 hundred thousands.

Sum, 699678 *Answer.*

In practice we shorten the process thus:

1234
567
8638= 7 times 1234.
7404 = 60 times 1234
6170 =500 times 1234
699678=567 times 1234. *Ans.*

Hence—*If the multiplier contains more than one figure, multiply by each figure separately, placing the right hand figure of each partial product under its own multiplier. The sum of these partial products will be the whole product.*

Proof.—Multiply the multiplier by the multiplicand, and see if the same result is obtained.

EXAMPLES.

	(2)	(3)	(4)
Multiply	5397	95375	$125.84
by	8009	35	54
	48573	476875	50336
	43176	286125	62920
Prod.	43224573	3338125	$6795.36

5. Multiply 874 by 21. *Ans.* 18354.
6. " 294 by 57. *Ans.* 16758.
7. " 1646 by 365. *Ans.* 600790.
8. " 8432 by 635. *Ans.* 5354320.
9. " 83×25 by 12×8. *Ans.* 199200.
10. " 83×12 by 25×8.
11. " 83×25×8 by 12.
12. " 25×8×83 by 12.

13. If there are 365 days in each year, how many days has a man lived when he is 57 years old?

Ans. 20805 days.

14. If a locomotive runs 25 miles an hour, 12 hours in a day for 304 days, how many miles does it run in that time? *Ans.* 91200 miles.

SECTION VIII.

TO MULTIPLY, MENTALLY, ANY NUMBER BETWEEN 12 AND 20, BY ANY NUMBER NOT EXCEEDING 12.

1. Multiply 17 by 6

42 = 6 times 7
6 = 6 times 10
102 = 6 times 17.

Multiply the units together; set down the right hand figure (2), and carry the tens' figure in the product to the multiplying figure. (4+6)=10, the result is the number of tens, to be placed on the left of the units. 6 times 7 is 42. 4 and 6 are 10 (tens).

2. Multiply 13 by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.
13×7=91; 13×9=117; 13×12=156.

3. Multiply 14 by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.
14×4=56; 14×6=84; 14×12=168.

4. Multiply 15 by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.
15×6=90; 15×9=135; 15×12=180.

5. Multiply 16, 17, 18, 19, by each of the numbers 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.

6. What cost 17 hundred weight of pork at 7 dollars a hundred weight?

7. If 9 men can do a certain work in 18 days, in how many days can 1 man do the same?

8. At $6 a barrel, what cost 19 barrels of flour.

9. At $7 a hundred, what cost 14 hundred feet of boards? 16 hundred feet?

10. If a stage coach go 6 miles an hour, how far will it go in 13 hours? 17 hours? 19 hours?

SECTION IX.

TO MULTIPLY MENTALLY, ANY TWO NUMBERS BETWEEN 12 AND 20.

1. Multiply, 17
by 16
42
6
7
10
Ans. 272

Here first we have 42, 2 units and 4 tens. Besides the 4 tens, there are (6+7+10) tens. Hence, Multiply the unit figures together, set down the units of the product, and add the tens in the partial product to both the unit figures, and increase the sum by 10; place the result on the left of the unit figure first obtained.

2. How many is 13×16? 3×6 contains 8 units, and 1 ten to add to (3, 6, and 10) tens, making 20 tens and 8=208. *Ans.*

3. How many is 17×19? 32 tens and 3=323. *Ans.*

4. Multiply 13 by 13, 14, 15, 16, 17, 18, 19, 20, 13×13=169; 13×17=221; 13×19=247.

5. Multiply 14 by 13, 14, 15, 16, 17, 18, 19, 20. 14×14=196; 14×18=252.

6. Multiply 15, 16, 17, 18, 19, by 13, 14, 15, 16, 17, 18, 19.

SECTION X.

APPLICATIONS OF THE LAST TWO SECTIONS.

1. At 15 cents a yard for calico, what cost 8 yards? 9 yards? 13 yards? 17 yards? 19 yards.

2. If 17 men build a wall in 14 days, in how many days could 1 man build it?

3. Paying 13 cents a piece for brooms, how much would 6 brooms cost? 9? 10? 14? 17? 19?

4. At 7 cents a yard for ribbon, what cost 15 yards? 16 yards? 17? 19? 20 yards?

5. If a steamboat moves 16 miles an hour, how far will it move in 6 hours? 9 hours? 12 hours? 16 hours? 19 hours?

6. At 13 dollars a ton, what cost 3 tons of hay? 7 tons? 12 tons? 13 tons? 16? 18? 19 tons?

7. Two men start from the same point and move in opposite directions, one 8 miles an hour, the other 9; how far apart will they be in 2 hours? in 3 hours? 5 hours? 9? 12? 14? 17? 18? 19? 20?

8. A train of cars moving 25 miles an hour, passes a stage coach going 8 miles an hour, in the same direction how far ahead of the stage will the cars be in 1 hour? in 2 hours? in 7 hours? 12? 13? 15? 18?

9. If I buy 8 tons of hay at 12 dollars a ton, and sell 5 tons at 15 dollars a ton, and 3 tons at 11 dollars a ton, how much shall I gain?

Solution. $8\times12=96$ *dollars cost.*

$5\times15=75$; $3\times11=33$: $75\times33=108$ dollars received.

$108-96=12$ *dollars gained. Ans. But, better thus;*

15 − 12 = 3 *dollars gain on each of* 5 *tons, makes* 15 *dollars gain.*

12 − 11 = 1 *dollar loss on each of* 3 *tons, makes* 3 *dollars loss.*

15 − 3 = 12 *dollars excess of gain over the loss.*

Ans. 12 *dollars.*

10. Bought 15 pounds of butter at 14 cents a pound, and 17 pounds at 18 cents a pound, and sold the whole at 19 cents a pound, how much was the gain? *Ans.* 75 + 17 = 92 cents.

If the whole had been sold at 17 *cents a pound, what would have been the gain or loss?*

Ans. 28 *cents gain.*

If the whole had been sold at 16 *cents a pound, what would have been the gain or loss?*

Ans. 4 *cents loss.*

11. If one man works 12 months for 14 dollars a month, and another for 19 dollars a month, how much more does the second man earn than the first?

Ans. 60 dollars.

12. What is the amount of two men's wages for a year, or 12 months, the one at 14 dollars a month and the other at 16 dollars.

Ans. $(14 + 16) \times 12 = 30 \times 12$.

13. A man made 14 pounds of maple sugar, worth 15 cents a pound, and mixed with it 16 pounds of Muscovado sugar at 8 cents a pound, and sold the whole at 15 cents a pound; how much did he gain by the mixing.

14. The interest of one dollar for one year in New York, and Michigan is 7 cents; how much is the interest of 2 dollars for the same time? How much is the interest of 12 dollars? of 15 dollars? of 19 dollars?

15. What is the interest of 3 dollars for 2 years? 4 dollars for 3 years? 7 dollars for 2 years?

SECTION XI.

TO MULTIPLY BY 10, 100, &c.; THAT IS, BY 1, WITH CIPHERS ANNEXED.

Annex to the multiplicand as many ciphers as there are ciphers in the multiplier.

NOTE.—To annex is to place after, or at the right hand; to prefix is to place before, or on the left.

When there are ciphers on the right hand of either or both the factors.

I.—*Place the numbers so that the right-hand significant figures shall stand under each other, and the ciphers at the right hand.*

II.—*Multiply the significant figures together, and annex to their product as many ciphers as there are on the right hand of both the factors.*

1. Multiply 250, by 10, 100, 1000, 10000.

Ans. 2500, 25000, 250000, 2500000.

2. Multiply 370 by 20, 200, 2000.

Ans. 7400, 74000, 740000.

3. If each soldier is an expense to the country of 150 dollars a year, what is the expense of 100 soldiers? 1000? what would be the expense of an army of 50,000 men? *Ans.* to the last, 7500000 dollars.

4. What is the worth of 120 acres of land at 45 dollars an acre? *Ans.* 5400 dollars.

5. How many hills of corn in 110 rows; each row containing 350 hills? *Ans.* 38500 hills.

6. How many letters are there in a book of 400 pages, having 50 lines on each page, and 60 letters in each line?

7. How many persons can be seated in a room having 68 seats, each of which will accomodate 120 persons? *Ans.* 8160.

8. If 50 cars, each car carrying 50 passengers, pass daily over a certain road, how many passengers are carried?

9. How many wagons, each carrying a ton, would furnish freight for 100 canal boats, each receiving 100 tons?

SECTION XII.

QUESTIONS IN MULTIPLICATION.

1. At 6 cents a pound for veal, what cost 23 pounds? 28 pounds, 34 pounds? 37 pounds?
Ans. $1.38; $1.68; $2.04; $2.22.

2. At 3 dollars a ream for paper, what cost 25 reams? 36 reams? 47 reams?

3. At 3 cents a mile, what must a passenger pay for riding 56 miles? 67 miles? 78 miles? 94 miles?

4. If a ship sails 7 miles an hour, how far will it sail in 24 hours? in 3 days and nights; that is, in 3×24 hours? in 4 days and nights?

5. At 9 cents a yard for cotton cloth, what will 1 piece cost, containing 31 yards? What will 3 such pieces cost? *Ans.* $8.37.

6. At 6 cents a pound for nails, what cost 15 pounds? 25 pounds? 56 pounds? 78 pounds?
Ans. 90 cents; $1.50; $3.36; $4.68.

7. At 7 cents a pound for lead, what will 34 pounds cost? 38 pounds? 47 pounds? 73 pounds?

Ans. $2.38; $2.66; $3.29; $5.11.

8. At 67 dollars a head, what will 3 horses amount to? 5 horses? 7 horses? 12 horses?

9. In 9 loads of oats, each containing 57 bushels, how many bushels? How many bushels in 12 loads?

10. At 12 dollars an acre for land, what will 58 acres cost? 67 acres? 86 acres?

Ans. $696; $804; $1032.

11. What cost 65 barrels of flour, at $7.25 a barrel? *Ans.* $471.25.

12. What cost 125 acres of land, at $37 an acre?

Ans. $4625.

13. What cost 148 yards of broadcloth, at $2.28 a yard? *Ans.* $337.44.

14. What cost 48 cords of wood at $2.75 a cord?

Ans. $132.

15. If an acre of land yields 57 bushels of corn, how many bushels are obtained from 36 acres?

Ans. 2052.

16. How many gallons are contained in 106 hogsheads, each hogshead containing 63 gallons?

Ans. 6678.

17. In 1 mile are 320 rods; how many rods in 40 miles? *Ans.* 12800.

18. In 1 quire of paper are 24 sheets; how many sheets in a ream of 20 quires? *Ans.* 480.

CHAPTER V.

DIVISION.

SECTION I.

1. How many oranges, at 4 cents apiece, can you buy for 12 cents?

Solution.—1 orange costs 4 cents; how many oranges cost 12 cents?

As many as the number of times that the price, 4 cents, is contained in 12 cents, which is 3 times. *Ans.* 3 oranges.

Proof.—3 oranges, at 4 cents apiece, cost 3 times 4 cents, 12 cents. The pupil should give the reasoning process on each question, till he can give it readily.

2. If 3 oranges cost 12 cents, what will 1 orange cost?

Solution.—Since 3 oranges cost 12 cents, 1 orange costs one third of 12 cents; as many cents as the number of times that 3 is contained in 12, which is 4 times. *Ans.* 4 cents.

3. At 6 cents a pound for veal, how many pounds can you buy for 42 cents? for 24 cents? for 54 cents?

4. If 7 pounds of veal cost 42 cents, what will 1 pound cost?

5. How many pairs of shoes, at $3 a pair, can you buy for $24?

6. If a man travels 45 miles in 9 hours, how far does he go in 1 hour?

7. If you pay $30 for 6 reams of paper, what is the price of 1 ream?

8. If 1 man can do a certain work in 20 days, in how many days can 4 men do it?

9. How many yards of cloth, at $3 a yard, can be bought for $15?

10. If $18 will pay for 6 yards of cloth, what is the price of 1 yard?

The process of finding how many times one number is contained in another, is called *Division.*

The number to be divided is the *Dividend.*

The number to divide by is the *Divisor.*

The result is the *Quotient:* it shows how many times the divisor is contained in the dividend.

The part of the dividend, less than the divisor, sometimes left after division, is the *Remainder.*

Ex.—4 is contained in 23 5 times, and 3 remains. Here 4 is the divisor, 23 the dividend, 5 the quotient, and 3 the remainder.

The short horizontal line between two dots ($\div$) is the sign of division, as it shows that the number on the left of it is to be divided by the number on the right; as $12 \div 4 = 3$. Twelve divided by 4 is equal to 3; or 4 is in 12 3 times.

When the divisor is 1, *the quotient is the same as the dividend; as,* 1 *is in* 5, 5 *times.*

When the divisor is *greater* than 1, the quotient is as many times *less* than the *dividend,* as, 4 in 12, 3 times, where 3 is as many times less than 12, as 4 is greater than 1.

When the divisor is a fraction, *less than* 1, the quotient is as many times *greater* than the dividend; as, one fourth of a unit is contained in 1, 4 times; in 2, 8 times; in 3, 12 times.

Division may be performed by successive subtractions; as, How many times is 4 *contained in* 12? *Subtracting* 4 *from* 12, *we have the remainders,* 8, 4, 0; *three subtractions, and* 0 *remains. Thus, division is a short way of performing several subtractions.*

11. If 2 quarts of strawberries cost 12 cents, what is the price of 1 quart? At 6 cents a quart for strawberries, how many quarts can you buy for 12 cents?

12. At 12 cents a yard for cambric, how much can you buy for 24 cents?

13. How many dozen eggs, at 12 cents a dozen, can you buy for 48 cents?

14. At 9 cents a yard for calico, how much can you buy for 27 cents?

15. If 5 pencils cost 30 cents, what is the price of 1 pencil?

16. At 6 cents apiece for pencils, how many can you buy for 30 cents?

17. At 48 cents for 6 yards of ribbon, what is the price of 1 yard?

18. At 8 cents a yard for ribbon, how much can you buy for 48 cents?

19. James paid 56 cents for 8 pine apples; what was the price of 1 pine-apple?

20. Paying 7 cents apiece for pine apples, how many can you buy for 56 cents?

21. Mary paid 28 cents for thimbles at 4 cents apiece; how many did she buy?

22. Jane paid 42 cents for primers at 6 cents apiece; how many did she buy?

23. If 6 yards of calico cost 54 cents, what is the price of 1 yard?

24. 63 is how many times 7? how many times 9?

Division is the reverse of multiplication. *The dividend is a product—the divisor is one of its factors, given; and the quotient is the other factor, sought.* This is seen in the following table, which is the multiplication table reversed. The pupil should become perfectly familiar with it.

DIVISION TABLE.

	1	2	3	4	5	6	7	8	9
2)	2	4	6	8	10	12	14	16	18
3)	3	6	9	12	15	18	21	24	27
4)	4	8	12	16	20	24	28	32	36
5)	5	10	15	20	25	30	35	40	45
6)	6	12	18	24	30	36	42	48	54
7)	7	14	21	28	35	42	49	56	63
8)	8	16	24	32	40	48	56	64	72
9)	9	18	27	36	45	54	63	72	81
10)	10	20	30	40	50	60	70	80	90
11)	11	22	33	44	55	66	77	88	99
12)	12	24	36	48	60	72	84	96	108
	1	2	3	4	5	6	7	8	9

Read the table 2 in 2 once; 2 in 4 twice; 2 in 6 three times, &c.

Taking the *dividend* 2, 3, 4, or any number of times *greater*, the *quotient* becomes as many times *greater;* as, 2÷2=1; 4÷2=2; 6÷2=3, &c., through the table.

Taking the *divisor* 2, 3, 4, or any number of times *greater*, the *quotient* becomes as many times *less;* as, 32÷2=16; 32÷4=8; 32÷8=4; 32÷16=2.

Taking both divisor and dividend any number of times greater or less, the quotient remains unchanged; 4÷2=2; 8÷4=2; 12÷6=2; 16÷8=2.

1÷½=2; 2÷½=4; 3÷½=6; that is

A *part* of a unit is contained in a whole number more times than a *unit* is contained in it; a half, twice as many times; a third, three times; a fourth, four times, &c., as,

To how many persons would $5 be distributed, if each person received ¼ of a dollar?

Ans. 4 persons to each dollar; 4×5=20 persons.

SUMMARY OF THE PRINCIPLES OF DIVISION.

Division is seeking for a multiplier, which being multiplied into the divisor will produce the dividend.

Multiplying the dividend, or *Dividing the divisor*, } *multiplies the quotient.*

Dividing the dividend, or *Multiplying the divisor*, } *divides the quotient.*

Multiplying or dividing both Dividend and divisor by the same number } *does not affect the quotient.*

SECTION II.

1. If 4 lead pencils cost 20 cents, what will 1 pencil cost?

Solution.—One pencil will cost one fourth as much as 4 pencils. If 4 pencils cost 20 cents, 1 pencil will cost one fourth part of 20 cents, which is 5 cents; for, 4 is in 20, 5 times.

2. Buying pencils at 5 cents a piece, how many can you buy for 20 cents?

Solution.—The 20 cents will buy as many pencils as the number of times that the price of 1 pencil is contained in 20 cents, 5 is in 20, 4 times. *Ans.* 4 pencils.

3. At 6 cents a quart for cherries, how many quarts can you buy for 12 cents? for 24 cents? for 18 cents?

4. At 4 cents a piece for oranges, how many can you buy for 16 cents? 12 cents? 8 cents?

5. What is the price per yard of ribbon, if you buy 3 yards for 21 cents? 4 yards for 28 cents? 2 yards for 14 cents.

6. If a boat moves 8 miles an hour, how long will it be in going 32 miles? 16 miles? 24 miles?

7. If you set trees, 9 trees in a row, how many rows will 36 trees make? 27 trees? 18 trees.

8. Ten cents make a dime; how many dimes in 40 cents? 30 cents? 20 cents?

9. How many miles an hour does a steamboat move, if it goes 33 miles in 3 hours? 22 miles in 2 hours? 44 miles in 4 hours?

10. What is the price per dozen for eggs, if 4 dozen cost 48 cents? if 3 dozen cost 36 cents? if 2 dozen cost 24 cents?

11. A man bought 4 pounds of meat, paid 37 cents, and received 5 cents in change; how much a pound did he pay for the meat?

12. Mary bought 3 yards of cambric, paid 50 cents, and received 14 cents in change; how much a yard was the cambric?

13. Four chandeliers have 28 lamps lighted, and 4 lamps not lighted; how many lamps are there in each of the 4 chandeliers?

SECTION III.

FRACTIONS.

James divided 5 oranges equally between his two sisters; how many oranges did he give to each?

Solution.—2 is in 5, 2 times, and 1 remains undivided. After giving them 2 oranges each, he must divide 1 orange into 2 equal parts (halves), and give each one half.

Ans. Two and one half oranges.

2. Divide 8 apples equally among 3 boys; how many apples will each have?

3 is in 8, 2 times, and 2 apples remain undivided. Dividing each of the two apples into 3 parts (thirds), there will be 6 pieces; from which each of the three boys will have 2 pieces, as many as there were apples remaining over 3 times 2, or 6 apples. *Ans.* Two and two thirds apples.

When an orange, an apple, or any other unit is divided into *parts*, the result is called a *fraction*.

If a unit is divided into 2 equal parts, each is called a *half;* if into 3 parts, a *third;* if into 4 parts, a *fourth*, &c. Hence, halves, thirds, fourths, &c., and, in general, numbers denoting *parts*, are *units of an order inferior to whole numbers*, and may be called *fractional units*, while *whole numbers* are *integral units*, or *integers*.

A *common* or *vulgar fraction* is an expression for any part or number of parts of a whole number, written with two numbers, one above the other, and a line between them; as, $\frac{5}{7}$ (*five sevenths*).

The lower number shows into how many parts the unit is divided; it gives name to the parts, and is therefore called a *denominator;* it shows the *denomination* of the parts.

The upper number shows how many of the equal parts are expressed in the fraction; it *numbers* the parts, and is therefore called a *numerator;* as, $\frac{5}{8}$ shows that a unit is divided into 8 parts, and 5 of those parts are expressed.

When the numerator is *less* than the denominator, the fraction is less than 1, and is called a *proper fraction;* as, $\frac{3}{7}$ (three sevenths).

When the numerator *equals* or *exceeds* the denominator, the value of the fraction equals or exceeds a unit; and the expression is called an *improper fraction;* as, $\frac{4}{4}$, $\frac{7}{5}$.

The numerator and denominator are the *terms* of the fraction.

A *mixed number* is composed of a whole number and a fraction; as, $5\frac{1}{2}$ (five and one half).

A whole number may be written in the form of a fraction, by writing 1 for its denominator; as, $3=\frac{3}{1}$ (*three ones*).

If 5 boys share 1 orange equally, each will have $\frac{1}{5}$; if they share 2 oranges, twice as much, $\frac{2}{5}$; for, 2 oranges being divided into 5ths, would make 10 pieces, which would be 2 pieces to each of the 5 boys; therefore, $\frac{2}{5}=2\div5$, $\frac{3}{5}=3\div5$; that is,

A fraction expresses division; the numerator is the dividend; the denominator is the divisor; and the value of the fraction is the quotient.

Therefore, $\frac{10}{5}=2$; $\frac{16}{5}=3\frac{1}{5}$; $\frac{27}{7}=3\frac{6}{7}$, &c., that is,

An *improper fraction* is reduced to a whole or mixed number, *by dividing the numerator by the denominator.*

The fraction is a sign of *division*, and may at any time be used instead of $(\div)$; as $\frac{12}{4}=12\div4=3$.

1. Divide each number from 1 to 24, by 2.

Ans. $1\div2=\frac{1}{2}$; $2\div2=1$; $3\div2=1\frac{1}{2}$; $4\div2=2$, &c.

One divided by 2 is one half; 2 divided by 2 is 1, &c.

2. Divide each number from 1 to 36, by 3.
3. Divide each number from 1 to 48, by 4.
4. Divide each number from 1 to 60, by 5.
5. Divide each number from 1 to 72, by 6.
6. Divide each number from 1 to 84, by 7.
7. Divide each number from 1 to 96, by 8.
8. Divide each number from 1 to 108, by 9.

9. Divide each number from 1 to 120, by 10.
10. Divide each number from 1 to 132, by 11.
11. Divide each number from 1 to 144, by 12.

SECTION IV.

1. In an orchard there are 28 trees in 7 rows; how many trees are there in each row?

2. A man bought 12 sheep for 78 dollars; how much a head did he give for them? *Ans.* 6½ dollars.

3. At 5 cents apiece, how many oranges can you buy for 40 cents?

4. What is the price per yard of calico, when 3 yards cost 36 cents? when 7 yards cost 84 cents?

5. At 7 cents a pound for beef, how many pounds can you buy for 28 cents? for 38 cents? for 58 cents?
Ans. 4 pounds; 5⅐ pounds; 8⅐ pounds.

6. If 4 stage coaches carry 36 passengers, how many passengers are there in each coach?

7. A man, having 1 dollar, bought 9 pounds of beef, and had 19 cents left; how much a pound did he pay for the beef?

8. A man paid 63 dollars for 7 tons of hay; how much a ton was the hay?

9. If 2 shillings will buy 1 yard of cloth, how many yards will 7 shillings buy?

10. If a carriage moves 5 miles an hour, in how many hours will it go 48 miles?

11. Four pecks make 1 bushel; how many bushels will fill a peck measure 30 times? 32 times? 42 times?

12. Eight furlongs make a mile; how many miles are there in 32 furlongs? 36 furlongs? 41 furlongs?

13. If a man earn 7 dollars a week, in how many weeks will he earn 56 dollars? 84 dollars?

14. If a man earn 84 dollars in 12 weeks, how much will he earn in 1 week?

15. At 2 cents apiece for apples, how many can you buy for 3 cents? for 4 cents? for 7 cents?

16. If a yard of cloth is worth 4 dollars, what is 1 fourth of a yard worth? 1 third of a yard?

Ans. 1 dollar; $1\frac{1}{3}$ dollars.

17. If 3 shillings will buy a bushel of corn, how much will 1 shilling buy? 2 shillings?

18. What are the factors of 72?

Ans. 8×9; 6×12; $3\times8\times3$; $3\times2\times4\times3$; or, $3\times2\times2\times2\times3$.

These small factors are found by first separating the number into any two factors; as, 8×9; then separating those into other smaller ones: thus, $8=2\times2\times2$; and $9=3\times3$; therefore $8\times9=2\times2\times2\times3\times3=72$.

19. What are all the small factors of 48?

20. How many factors greater than 1 has 56?

Ans. 4. Find them.

21. Find the four factors of 24.

22. Find the four factors of 90.

23. Find the four factors of 54.

24. Find the three factors of 63.

25. Find the five factors of 108.

SECTION V.

A number composed of two or more whole numbers greater than 1, as factors, is a *composite number;* as, 6, which is 3×2; and 12, which is $2\times2\times3$.

A number which is not composite is called a *prime number;* as, 2 and 3 are prime numbers, and are also the *prime factors* of 6.

1. Find the prime factors of 60, 80, and 90.
2. Find the prime factors of 84, 18, and 66.
3. Find the prime factors of 120, 132, and 121.
4. Find the prime factors of 36, 144, and 72.
5. Find the prime factors of 112, 200, and 300.
6. If 8 is one factor of 9, what is the other factor? *Ans.* $1\frac{1}{8}$.
7. 12 is how many times 11? *Ans.* $\frac{12}{11}$ or $1\frac{1}{11}$.
8. 11 is how many times 12? *Ans.* $\frac{11}{12}$.
9. 18 is how many times 3? how many times 5?
10. 48 is how many times 3 times 4? *Ans.* 4.
11. If 9 is a factor of 15, what is the other factor?
12. James, having 18 dollars, has 4 times as many as William; how many dollars has William?
13. A man laid out 17 dollars for cloth at 5 dollars a yard; how many yards did he buy?
14. At 5 dollars a yard for cloth, how much can you buy for 1 dollar? for 2 dollars? 3 dollars? 6 dollars? 10 dollars?
15. What is the price per barrel for flour, when you pay 14 dollars for 2 barrels? 21 dollars for 3 barrels?
16. Peter and John went to market together. Peter paid 56 cents for 8 pounds of beef, and John bought 1 pound. How much should John pay?
17. What cost 8 pounds of beef at 7 cents a pound?

SECTION VI.

The money value of a unit of any commodity, as 1 pound, 1 yard, &c., is called *price;* and as a distinctive term, we will call the value of any number more or less than 1, the *cost;* and the number of things whose cost is estimated, the *quantity;* as,

Quantity.		*Price.*		*Cost.*
4 pounds	at	10 cents per lb.	=	40 cents.
½ pound	at	10 cents per lb.	=	5 cents.

The cost = the price × the quantity; $40 = 4 \times 10$.
The price = the cost ÷ the quantity; $10 = \frac{40}{4}$.
The quantity = the cost ÷ the price; $4 = \frac{40}{10}$.
Hence the following

USEFUL RULES.

I.—To find the cost of goods,
Multiply the price by the quantity.
II.—To find the price,
Divide the cost by the quantity.
III.—To find the quantity,
Divide the cost by the price.

The *cost* will always be of the same name as the price. The multiplier is always an abstract number; and the product is of the same name as the multiplicand.

Therefore, since division is reversing the process of multiplication,

The divisor and quotient will be, one of them, an abstract number, and the other will have the same name with the dividend.

1. If 4 tons of hay cost 48 dollars, what is the price per ton?

3. At 12 dollars a ton for hay, what quantity can you buy for 48 dollars?

3. What cost 4 tons of hay when the price is 12 dollars a ton?

4. What will 9 hundred weight of beef come to, at 7 dollars a hundred weight?

5. Paid 63 dollars for 9 hundred weight of beef, what was the price per hundred?

6. Paid 63 dollars for beef at 7 dollars a hundred weight; how much did it weigh?

7. At 8 cents a pound for sugar, what will 12 pounds cost?

8. At 8 cents a pound, how much sugar will 96 cents pay for?

9. If 12 pounds of sugar cost 96 cents, what is the price per pound?

10. At 12 cents a yard for sheeting, what will $7\frac{1}{2}$ yards cost?

11. At 12 cents a yard, how much sheeting can you buy for 90 cents?

12. At $7\frac{1}{2}$ cents a pound for cheese, what will 6 pounds cost?

Solution.—6 pounds at 7 cents=42 cents.
6 pounds at $\frac{1}{2}$ cent = 3 cents.
By adding, 6 pounds at $7\frac{1}{2}$ cents=45 cents. *Ans.*

13. If 6 pounds of cheese cost 45 cents, what is the price per pound?

14. What is $\frac{2}{3}$ of 7?

$\frac{2}{3}$ of $7=\frac{1}{3}$ of 2 times $7=14\div 3=4\frac{2}{3}$. *Ans.*

That is—*To take any fractional part of a number, multiply by the numerator, and divide by the denominator.*

Divide first by the denominator, when you can do

this without a remainder; as, $\frac{1}{3}$ of 15 is what number? $15 \div 3 = 5$; $5 \times 2 = 10$. *Ans.*

15. At $12\frac{1}{2}$ cents a pound for butter, what will 7 pounds cost?

16. If 7 pounds of butter cost $87\frac{1}{2}$ cents, what is the price per pound?

$87\frac{1}{2} \div 7 = 12$, and $3\frac{1}{2}$ remain.

$3\frac{1}{2} \div 7 = 7 \div 14 = \frac{1}{2}$. *Ans.* $12\frac{1}{2}$ cents.

SECTION VII.

When the dividend is more than ten times the divisor,

Divide the tens in the dividend, for the tens in the quotient; then divide the tens that remain, if any, with the units, for the units in the quotient.

1. Divide 162 cents equally among 3 boys; how many cents shall each have?

162=16 tens and 2.
16 tens÷3=5 tens, and 1 ten remains; therefore,
162÷3=5 tens, and 12 units remain.
12÷3=4. *Ans.* 54 cents.

2. If 3 boys receive 54 cents each, how much will they all receive?

3. When railroad fare is 3 cents a mile, how far can you ride for $2.79?

4. At 4 cents a mile, how far can you ride in a stage coach for $2.64?

5. At 1 third of a dollar a bushel, what will 84 bushels of potatoes cost? 95 bushels? 105 bushels?

6. Paying 1 dollar for 4 yards of cloth, what will 68 yards cost? 79 yards? 125 yards?

Ans. to the last, $31\frac{1}{4}$ dollars, or $31.25.

7. What number multiplied by 9 will make 324?
8. How many times 7 is 245? *Ans.* 35.
9. How many times 8 is 736?
10. 335 is 5 times what number? *Ans.* 67.
11. 960 is 3 times what number?
12. When sugar is 11 cents a pound, how much can you buy for $3.85?
13. What cost 35 pounds of sugar at 11 cents a pound?
14. How many weeks are there in 324 days? in 242 days? in 365 days?
15. How many days are there in 46 weeks, 2 days? in 34 weeks, 4 days? in 52 weeks, 1 day?
16. How many barrels of flour, at $9 a barrel, can you buy for $135?
17. How many hats, at $3 apiece, can you buy for $45?

SECTION VIII.

TO DIVIDE A LARGE NUMBER.

First when the divisor does not exceed 12.

1. If 18707 cents, or $187.07 are equally divided among 5 persons, how much will each receive?

Operation.
5)187.07
37.41 2/5
Proof 187.07

For convenience, place the divisor on the left of the dividend. Each person must have 1 fifth part of the dividend. Since 5 is not contained in 1 (*hundred*), there can be no hundreds in the quotient, 5 is in 18, (*tens*) 3 (*tens*) times, and 3 (*tens*) remain. Write 3 under the 8. The 3

tens remaining must be divided with the next figure 7 units, making 37 units of dollars, 5 in 37, 7 times, and 2 remains; each one now has 37 dollars; the 2 remaining dollars must be divided with the next figure, 0, (*dimes*), making 20 (*dimes*), 5 in 20, 4 times; write 4 under the 0 dimes. Now, since no dimes remain, divide the 7 (*cents*,) alone; 5 in 7, 1 time, and 2 remains. Write 1 under the 7, and finally divide the 2 by 5, making the fraction $\frac{2}{5}$. Hence the Answer, $3741\frac{2}{5}$ cents, or $\$37.41\frac{2}{5}$.

When the divisor does not exceed 12, the process is called *Short Division*, in which observe the following

RULE.

I.—*Place the divisor on the left of the dividend, separate them by a curved line, and draw a right line under the dividend.*

II.—*Seek how many times the divisor is contained in the fewest figures of the dividend that will contain it, and place the result under the last figure taken, for the first figure of the quotient.*

III.—*If there is no remainder, divide the next figure of the dividend, but if there is a remainder, conceive it to be prefixed to the next figure of the dividend before making the division. If any part of the dividend to be divided is less than the divisor, write* 0 *in the quotient, and consider that part as a remainder.*

If there is a final remainder, write it over the divisor to form a fraction, and join it to the quotient.

Proof.—Multiply the quotient by the divisor, add in the remainder, and the result will be equal to the dividend.

EXAMPLES.

(2)	(3)	(4)
5)3256275	6)7236012	7)182343
651255	1206002	26049

5. Divide 46240 by 3; by 4, 5, 6, 7, 8, 9, 11, and 12.

Answers, 15413⅓; 11560, 9248, 7706⅔, 6605⅐, 5780, 5137⅞, 4203 7/11, 3853 4/12.

6. A has 20160 dollars. B has ½, C ¼, D ⅛, E ⅙ and F ⅐ as much money as A. How much have all the six persons? *Ans.* 42192 dollars.

7. In the foregoing example, how many dollars have B, C, and D? How many have E and F?

Ans. B, C, and D, $15792; E and F $6240.

8. 8 times 320 is equal to how many times 4?

Ans. 620 times.

But this is known without performing the entire work thus. 8 times 6=4 times 12; 12 being twice 6 while 4 is half of 8. Therefore,

8 times 320=4 times 640: 640 being twice 320.

9. 8 times 320 is how many times 4?

8 times 320 is how many times 2?

10. How many barrels of flour, at $6 a barrel, can you buy for $1944? *Ans.* 324.

11. There are 7 days in 1 week; how many weeks in 14 days? in 21 days? in 728 days? *Ans.* 104.

12. When land is $9 an acre, how many acres can you buy for 18? for $1200? *Ans.* 133⅓.

13. If 6 yards of cloth make a suit of clothes, how many suits will 1953 yards make? *Ans.* 325½.

14. 83259 is how many times 9? *Ans.* 9251.

15. 7268 is how many times 7? *Ans.* 1038.

SECTION IX.

LONG DIVISION.

Long Division is when the divisor contains several figures.

1. At 27 dollars per acre, how many acres of land can be bought for 25542 dollars? *Ans.* 946 acres.

Div. Divid. Quotient.

27)25542(946
243=1st product
124=tens.
108=2d prod.
162=units.
162=3d prod.
000=rem.

Set the divisor and dividend as before, but the quotient on right, 27 (*dollars*) is contained in 255 (*hundreds of dollars*) 9 (*hundreds*) times; found by trial; write 9 for the first figure of the quotient. To know how much remains of the 255 (*hundreds*), multiply 27 by 9, and subtract the product from 255; $27\times9=243$. $255-243=12$ (*hundreds*), remaining; annexing the 4 tens in the dividend, we have 124 (*tens*), in which 27 is contained 4 (tens) times, for the next quotient figure. $27\times4=108$; $124-108=16$ (tens) remaining. Annexing the 2 units in the dividend, we have 162, in which 27 is contained 6 times; for, $27\times6=162$. Therefore, the quotient is 946 acres.

PROOF.

900 acres at	$27 per	acre—	243	hund. dol.—1st prod.
40 acres	"	"	— 108	tens of dol.—2d prod.
6 acres	"	"	— 162	dollars—3d product.
946 acres	"	"	—25542	dollars, by addition.

Hence, we infer the following

RULE FOR LONG DIVISION.

I.—*Place the divisor at the left of the dividend, draw a curve line between them, and another curve line on the right of the dividend. Seek how many times the divisor is contained in the fewest figures of the dividend that will contain it, and set the result at the right of the dividend for the first figure in the quotient.*

II.—*Multiply the divisor by this figure, subtract the product from the figures divided, and to the remainder annex the next figure of the dividend, for a new partial dividend.*

III.—*Divide this number as before, for the next quotient figure, multiply, subtract, annex the next figure of the dividend, divide the partial dividend so formed, &c., till all the figures of the dividend are annexed and divided, and the final remainder found, (if any), which place over the divisor to form a fraction;*

Proof of division.

1. To the product of the divisor and quotient, add the remainder; the result is equal to the dividend.

2. Subtract the remainder (if any) from the dividend, and divide by the quotient; the result will equal the divisor.

EXAMPLES.

2. Divide 315281 by 23. *Ans.* $13707\frac{20}{23}$.
3. Divide 826052 by 75. *Ans.* $11014\frac{2}{75}$.
4. Divide 146582 by 237. *Ans.* $618\frac{116}{237}$.
5. Divide 10323 by 37. *Ans.* 279.
6. Divide 10323 by 279. *Ans.* 37.

7. Divide 34314 by 798. *Ans.* 43.
8. Divide 42581 by 49. *Ans.* 869.
9. Divide 16758 by 57. *Ans.* 294.
10. Divide 26598 by 62. *Ans.* 429.
11. Divide the 7th, 8th, 9th, 10th, each by its quotient.

SECTION X.

When the divisor has ciphers on the right.

Cut off the ciphers from the right of the divisor, and cut off as many figures from the right of the dividend. Divide the remaining figures of the dividend by the significant figures of the divisor, and to the remainder annex the figures cut off from the dividend, for the true remainder.

1. Divide $2393 equally among 50 men: how many dollars will each receive?

Ans. 47\frac{43}{50}$, or $47.86.

Operation.

5|0)239|3 ·

47|4 rem.

Ans. 47$\frac{43}{50}$

The divisor is a composite number, 10×5. To divide by 10, cut off one figure; the quotient is 239, and 3 remains; dividing 239 by 5 the quotient is 47, and 4 remains. But this remainder is 4 tens, while the first is 3 units. Therefore annex the first remainder to the second; and the result 43 (4 tens and 3 units), is the true remainder.

EXAMPLES.

2. Divide 1078000 by 11000. *Ans.* 98.
3. Divide 40167 by 180. *Ans.* 223$\frac{127}{180}$.
4. Divide 876432 by 980. *Ans.* 894$\frac{312}{980}$.

5. Divide 732412 by 98700. *Ans.* 7, and 41512 rem.

6. Divide 97632 by 5300. *Ans.* 18, and 2232 rem.

7. Divide 25070 by 10, 100, and 1000.

Ans. 2507, $250\frac{70}{100}$, $25\frac{70}{1000}$.

In this case, we have only to cut off as many figures from the dividend at the right, as there are ciphers in the divisor, the remaining figures on the left are the quotient, and those at the right are the remainder.

8. Divide 352 by 10, and by 100.

Ans. $35\frac{2}{10}$ and $3\frac{52}{100}$.

9. How many boats are required, to carry 10000 tons of coal, if each boat carries 50 tons.

SECTION XI.

1. How many yards of cloth, at 25 cents a yard, can be purchased for $3?

Statement.—(1), 25 cents—price of 1 yard;
(2), 300 cents—cost of how many yards?

Equal quantities, multiplied by equal multipliers, make equal products. Therefore multiply both parts of the first equality by such multiplier that we shall have 300 cents, in place of 25 cents. What is that multiplier? It is the number of times that 20 is contained in 300, which is 12. Multiplying, we have the proof.

25×12 (cents)—cost of 1×12 (yards); or, 300 cents—cost of 12 yards. *Ans.* 12.

2. At $8.05 an acre, how many acres of land can be purchased for $161? *Ans.* 20 acres.

3. At $1.125 a yard, how many yards of cloth can be bought for $234? *Ans.* 208 yards.

4. If 15 firkins of butter, each containing 25 pounds, are sold for $60, what is the price per pound?

Ans. 16 cents.

5. If 27 acres of land cost $408.375, how much is the price per acre?

Statement.—$408.375—cost of 27 acres;
how much—price of 1 acre?

If we divide the terms of the statement by 27, we have $408.375÷27—cost of 27÷27—cost of 1 acre.

$408.375÷27—$15.125. *Ans.*

6. If 119568 men are divided into 106 regiments, how many men are in each regiment? *Ans.* 1128.

7. If a locomotive runs 97344 miles in 576 days, how far is that for each day? *Ans.* 169 miles.

SECTION XII.

1. If 8 times 3 is 24, 16 times 3 is how many times 24?

16 times 3 is 48, which is 2 times 24; 2 times 8 times 3—16 times 3—48. That is, 16 times 3 is twice as great a number as 8 times 3; because 16 is 2 times 8.

So also 15 times 328—5 times 3 times 328. That is,

Two numbers that consist of the same factors, are equal; and if one factor of a number is made 2, 3, *or any number of times greater or less, the product becomes as many times greater or less.*

2. If 6 times 9=54, 2 times 9=how many times 54?

3. If 8 yards of cloth cost 27 dollars, 16 yards will cost how many times 27 dollars? *Ans.* Twice.

4. If (See Ex. 3) 8 yards cost 27 dollars, (or 16

yards cost 2×27 dollars,) how many dollars will 16 yards cost?

5. 8 is how many times 3? The number of times that 3 is in 8, is $2\frac{2}{3}$ or $\frac{8}{3}$, or the third part of 8. Since the third part of 1 is $\frac{1}{3}$, the third of 2 is $\frac{2}{3}$, and the third of 8 is $\frac{8}{3}$.

6. 3 is how many times 8? 3, *which is less than* 8, *is a part of* 8. *Ans.* $\frac{3}{8}$.

7. 11 is what part of 9? 11 *is more than* 9; *therefore an improper fraction of* 9. *Ans.* $\frac{11}{9}$.

8. 9 is how many times 11, or what part of 11?

9. 27 is what part of 13?

10. 23 is how many times 17?

11. 17 is how many times 23?

12. If 17 dollars will buy a certain quantity of cloth, how many times the same quantity will 23 dollars buy? *Ans.* $\frac{23}{17}$.

13. If 51 yards of cloth cost 17 dollars, how much cloth can you buy for 23 dollars?

Solution.—23 *dollars will pay for* $\frac{23}{17}$ *as much as* 17 *dollars;* $\frac{23}{17}$ *of* 51 *yards* $=$ 23 *times* $\frac{1}{17}$ *of* 51 *yards* $=$ 23 *times* 3 *yards* $=$ 69 *yards. Ans.* Or,

If 17 *dollars buy* 51 *yards,* 1 *dollar will buy* $\frac{1}{17}$ *of* 51 *yards, which is* 3 *yards; and* 23 *dollars will buy* 23 *times as much as* 1 *dollar.*

23 *times* 3 *yards* $=$ 69 *yards. Ans.*

14. If 3 yards of cloth cost $4, what will 7 yards cost?

$\frac{7}{3}$ of $4, *which is the same as* 7 *times* $\frac{1}{3}$ *of* 1, *or* 4 *times* $\frac{7}{3}$ *of* $1 = \$\frac{28}{3} = \$8\frac{4}{3}$. *Ans.*

15. If a staff 4 feet high cast a shadow 7 feet long, how many times the same height is a steeple which casts a shadow 200 feet?

16. If a steeple is $\frac{200}{7}$ of 4 feet high, how many feet high is it? $800 \div 7 = 114\frac{2}{7}$ feet. *Ans.*

17. If 5 barrels of flour cost a certain sum, how many times that sum will 2 barrels cost? *Ans.* $\frac{2}{5}$ of it. Then if the 5 barrels cost $26, how much will 2 barrels cost? $\$26 \times 2 \div 5 = \$52 \div 5 = \$10.40$. *Ans.*

18. If 10 barrels of flour cost $52, what cost 2 barrels?

19. If 4 men do a piece of work in 5 days, in what part of 5 days would 3 men do it? If a number of men do a piece of work in $\frac{4}{5}$ of 5 days, in how many days would they do the whole?

20. If a quantity of provisions will serve 5 men 7 days, how many days would it serve 9 men?

SECTION XIII.

1. James and William bought 40 marbles, but James paid for 10 more of them than William; how many shall James have?

Solution.—Setting aside the 10 which belong to James alone, 40—10=30 belong equally to each, 30÷2=15, are William's, 15+10=25, are James'.

Proof, 15+25=40, the whole.

2. Henry and John, 50 rods distant from each other, set out at the same instant and ran to meet each other; on meeting, it was found that Henry ran 3 rods farther than John. How far did each run?

3. Henry, Peter, and Seth, have among them 86 cents, of which Peter has 5 more than Henry, and Seth has 7 more than Peter; how many has each?

Solution.—Set aside 5 cents for Peter, and 7 more than 5 making 12 for Seth; which are 17 for Peter and Seth. The remainder 86—17=69 cents, are shared equally by the

three. 69+3=23 cents for Henry. 23+5=28 for Peter; 28+7=35 for Seth.

4. A man undertook to cut a rope 100 feet long into 3 equal pieces, but found that the second piece was 4 feet longer than the first; and the third, 2 feet longer than the second; how long was the shortest piece?

5. Three equal pieces of land are together worth 13 dollars an acre; but the second is worth 2 dollars an acre more than the first; and the third 5 dollars an acre more than the second. How much an acre is each worth?

6. At a school district meeting, 64 men elected a trustee, 4 men were voted for; B had 2 more votes than A; C 3 more than B; and D 8 more than C. How many votes had each?

7. A B C and D built a wall together. A worked 1 day alone; then B commenced. When A and B had worked together 2 days, C commenced. When the three had worked together 3 days, D commenced, and the 4 worked together till the job was finished; when they found that 66 days' work had been done. How many days did each one work, and how much shall each receive, for wages, at $1\frac{1}{2}$ dollar a day?

Ans. A earned $28\frac{1}{2}$ dollars; B, 27 dollars; C, 24 dollars; and D $19\frac{1}{2}$ dollars.

8. If to-day you have 5 shillings, and to-morrow receive 7 shillings, and then pay out 3 fourths of your money, how much will you have left?

9. If to-day you have 12 shillings, to-morrow expend 3 shillings, and the next day expend 2 thirds of the remainder, how much will you have left?

10. If you have 50 dollars, and buy 5 barrels of flour at 7 dollars a barrel, and 2 yards of cloth at 8

dollars a yard, for how many days can you hire a carpenter at two dollars a day, with the remainder of the money? *Ans.* $4\frac{1}{2}$ days.

11. A man having 100 dollars paid 68 dollars for a horse, and half a dollar a week for 10 weeks, for pasturing him; he received 20 dollars for the services of the horse, and sold him for 90 dollars; how much was the gain or loss on the horse?

12. If the standard foot should be so enlarged, that what is now 7 feet, should become 6, what would 5 feet become? 1 *foot would become* $\frac{6}{7}$; *and* 5 *feet would become* 5 *times* $\frac{6}{7}=30\div 7=4\frac{2}{7}$. *Ans.*

13. If 3 bushels of wheat are equal in value to 7 bushels of oats; how many bushels of oats are equal to 7 of wheat? 7 *of wheat=how many times* 3 *of wheat?* $7\div 3=2\frac{1}{3}$.

Then $2\frac{1}{3}$ *times* 7=*how many? Ans.* $16\frac{1}{3}$ bushels.

SECTION XIV.

CANCELLATION.

Multiplication and *division* are the reverse of each other, so that, if we multiply and divide by the same number, the two operations *cancel* or destroy each other; as

1. 9 times 7 divided by 7, is 9; written $\frac{9\times 7}{7}=9$. This is the same as 9 times $\frac{7}{7}$, (seven 7ths), or $9\times 1=9$.

So also, a factor of the dividend may cancel the divisor or a factor of it; or a factor of either may cancel a factor of the other; as,

2. $\frac{7\times 8}{4}=\frac{7\times 2}{1}=14$; by cancelling 4; a factor of 8.

3. $\frac{9\times 7\times 8}{8}=\frac{9\times 7}{1}=\frac{63}{2}=31\frac{1}{2}$; by cancelling 4 from 28 and 8. By rejecting the factor 4, the divisor and dividend are each $\frac{1}{4}$ as great as before; but the quotient, which is the thing sought, is unchanged.

4. Divide the product of $12\times 35\times 8$ by 21×4.

$$\frac{\cancel{4}\quad 5}{\cancel{12}\times\cancel{35}\times 8}{\cancel{21}\times\cancel{4}}=40.$$
$$\cancel{3}$$

First cancel 7 from 35 and 21, 5 and 3 are left. Then 3 is in 12, 4 times; cancel 3, and take 4 instead of the 12, from which 3 is rejected. Then cancel the 4s, and we have $5\times 8=40$. If the factors of the dividend had all been cancelled, and a divisor had been left=40, the result would have been the fraction $\frac{1}{40}$.

When all the factors cancel each other, the result is $\frac{1}{1}=1$.

5. $3\times 2\times 5=30$: $\frac{3\times 2\times\cancel{5}\quad 30}{\cancel{5}}=\frac{}{5}=6.$

$\frac{3\times\cancel{2}\times\cancel{5}}{\cancel{2}\times\cancel{5}}=\frac{30}{2\times 5}=3.$

That is, cancelling a factor of any number, is dividing the number by that factor.

Cancelling two or more factors of a number, is dividing by the product of those factors.

The *quotient* consists of those factors of the *dividend*, that are *not contained* in the *divisor*. Hence,

When *multiplications*, or *divisions*, or *both*, are to follow each other,

Indicate by signs, the operations to be performed; cancel the factors common to the dividend and divisor, and employ the remaining factors.

NOTE.—1. A factor cancelled becomes equal to 1.

2. A factor cancels its equal but once.

3. A factor may cancel two or more others, whose *product* is equal to that one; as, $\frac{5\times\cancel{12}}{\cancel{3}\times\cancel{4}}=5$.

6. What is the product of $(77\times25\times7)\div(3\times25)$? *Ans.* 175.

$\frac{\cancel{77}\times25\times7}{\cancel{3}\times\cancel{25}}=25\times7=175.$ | $\cancel{3}\,\cancel{77}$ / $\cancel{25}\,25=175.$ / 7

In the use of blackboards, the *perpendicular* line may be used, setting the factors of the *dividend* on the *right*, and those of the *divisor* on the *left*.

7. If 14 pounds of beef cost 105 cents, what cost 48 pounds?

$\frac{15\quad 24}{\cancel{105}\times\cancel{48}}=15\times24=360$ cents.—*Ans*
$\cancel{14}$
$\cancel{2}$

One 14th part of 105 cents = price per pound, which multiplied by 48 = cost of 48 lbs.

Or, 48 times 105 cents = cost of 48 lbs., at 105 cents per lb., which would be 14 times too much; *Therefore, when a multiplication and a division are required, the result will be the same whichever is done first.*

8. When 80 eggs are sold for 1 dollar, what will 12 dozen cost? *Ans.* $1.80.

9. If 6 quarts of currants cost 27 cents, what cost 26 quarts? *Ans.* $1.17.

10. What is the value of $36\times48\times100$, divided by $12\times24\times200$? Divided by $9\times96\times10$? Divided by $72\times16\times10$? *Answers.* 3, 20, and 15.

11. $(56\times99\times8)\div(63\times88)$ = how many? *Ans.* 8.

12. Divide $(20\times6\times49)$ by $(24\times35.)$ *Ans.* 7.

13. Divide $(990\times50\times6)$ by (108×220).

Ans. $12\frac{1}{2}$.

14. If 5 men can do a piece of work in 12 days, in how many days will 3 men do it?

Solution.—3 *men must have* $\frac{5}{3}$ *as much time as* 5 *men.* $\frac{5}{3}$ *of* $12=5\times4=20$ *days. Ans.* by cancelling 3.

15. If 24 pounds of beef cost \$2.16, what will 23 pounds cost? 23 *pounds cost what part of the cost* 24 *pounds? Then* $\frac{23}{24}$ *of* $216=$*what number? Cancel* 24. *Ans.* \$2.07.

16. If 13 yards of cloth cost \$32.50, what will 14 yards cost? Here cancel 13. *Ans.* \$35.

17. If 90 bushels of oats feed a certain number of horses 6 days, how long will 450 bushels feed them?

Ans. 30 days.

18. If 12 yards of cloth cost \$48, how many yards can you buy for \$112? *Ans.* 28 yards.

19. If 15 bushels of corn are had in exchange for 60 bushels of potatoes, how much corn may be had for 140 bushels of potatoes? *Ans.* 35 bushels.

20. If 13 pounds of tea cost \$2.60 cents, what will 25 pounds cost?

21. If 15 sheep cost \$20, what will 36 sheep cost?

Ans. \$48.

22. The Russian mile contains 1100 yards, and the English mile 1760 yards; how many English miles are equal to 176 Russian?

23. If John runs 25 rods while Henry runs 20, how far must John run to gain 15 rods of Henry?

Ans. 75 rods.

24. If 42 cents buy 7 oranges, how many cents will 15 oranges cost?

CHAPTER VI.

FRACTIONS.

SECTION I.

1. If an apple is divided into 20 pieces, what fraction of an apple will 1 piece be? 3 pieces? 7 pieces? 11 pieces?

What fraction will 10 pieces be? $\frac{10}{20}$.

But ten twentieths ($\frac{10}{20}$) *of an apple are the same as* 5 *tenths; for every* 2 *twentieths will make* 1 *tenth.* $\frac{10}{20}=\frac{5}{10}$; *and* 5 *tenths of an apple are the same as* 1 *half; since* 5 *is half of* 10.

Therefore $\frac{10}{20}=\frac{5}{10}=\frac{1}{2}$. The numerator and denominator are called the *terms* of a fraction; and when the value of a fraction is written in the least numbers possible, the fraction is said to be in its *lowest terms.* The last example shows that *if both terms of a fraction are multiplied or divided by the same number, the value remains the same.* Hence to reduce a fraction to its lowest terms.

Divide both terms of the fraction by any number that will divide them, and the quotients again in the same manner, as far as possible.

2. Reduce $\frac{2}{4}$, $\frac{8}{16}$, $\frac{20}{40}$, $\frac{50}{100}$, $\frac{15}{30}$, and $\frac{160}{320}$, each to its lowest terms. *Ans.* $\frac{1}{2}$.

3. Reduce $\frac{6}{12}$, $\frac{10}{18}$, $\frac{14}{21}$, $\frac{18}{27}$, $\frac{24}{36}$, $\frac{24}{64}$. *Ans.* $\frac{2}{3}$.

4. Reduce $\frac{10}{14}$, $\frac{15}{21}$, $\frac{20}{28}$, $\frac{25}{35}$, $\frac{40}{56}$. *Ans.* $\frac{5}{7}$.

5. Reduce $\frac{8}{16}$, $\frac{12}{27}$, $\frac{16}{30}$, $\frac{24}{54}$, $\frac{28}{63}$. *Ans.* $\frac{4}{9}$.
6. Reduce $\frac{18}{24}$, $\frac{27}{36}$, $\frac{26}{52}$, $\frac{48}{64}$, $\frac{54}{72}$. *Ans.* $\frac{3}{4}$.
7. Reduce $\frac{26}{34}$, $\frac{39}{51}$, $\frac{52}{68}$, $\frac{65}{85}$, $\frac{78}{102}$. *Ans.* $\frac{13}{17}$.

SECTION II.

1. In $\frac{23}{3}$ (23 thirds) of an orange, are how many oranges?

Since 3 *thirds of an orange make* 1 *whole orange,* 23 *thirds will make as many as the number of times that* 3 *is contained in* 23, *which is* $7\frac{2}{3}$ *times. Ans.* $7\frac{2}{3}$ oranges. Hence, to change an improper fraction to a whole or mixed number. *Divide the numerator by the denominator, and the quotient will be the whole or mixed number required.*

2. In $\frac{47}{4}$ of a dollar, how many dollars?
3. In $\frac{75}{4}$ of a yard, how many yards? *Ans.* $18\frac{3}{4}$.
4. A minute is $\frac{1}{60}$ of an hour: in 132 minutes, how many hours?
5. An inch is $\frac{1}{12}$ of a foot: how many feet in 147 inches?
6. How many feet in 200 inches?
7. In $\frac{247}{24}$ of a day, how many days? *Ans.* $10\frac{7}{24}$.
8. What is $\frac{17}{7}$? $\frac{10}{8}$? $\frac{13}{5}$?
9. 1 shilling is $\$\frac{1}{8}$: how many dollars in 100 shillings?
10. In $\frac{266}{60}$ of a month, how many months?
11. In $\frac{75}{3}$ of a yard, how many yards?
12. In $\frac{1000}{3240}$ of a mile, how many miles?
13. In $\frac{344}{60}$ of an hour, how many hours?
14. In $\frac{294}{7}$ of a week, how many weeks?
15. In $\frac{730}{365}$ of a year, how many years?

SECTION III.

1. In 7 oranges are how many thirds of an orange?

Since 1 *orange contains* 3 *thirds*, 2 *oranges contain twice* 3 *thirds;* 3 *oranges*, 3 *times* 3*; and* 7 *oranges*, 7 *times* 3 *thirds, which is* 21 *thirds*, $\frac{21}{3}=7$. *Ans. So also* $7\frac{2}{3}$ *would make* 21 *thirds and* 2 *thirds, or* $\frac{23}{3}$. Therefore,

To change a whole or mixed number to an improper fraction. *Multiply the whole number by the given denominator, and to the product add the numerator: write the result over the denominator.*

2. In $8\frac{3}{4}$ apples, how many fourths of an apple?
3. In $44\frac{1}{2}$ yards, how many half yards?
4. $16\frac{1}{2}$ feet=1 rod, make how many half feet?
5. $21\frac{17}{18}$ is what fraction? *Ans.* $\frac{395}{18}$?
6. What improper fraction is equal to $13\frac{12}{101}$? *Ans.* $\frac{1325}{101}$.
7. Change $25\frac{75}{100}$ to a fraction.
8. Change $34\frac{3}{12}$ to a fraction. *Ans.* $\frac{411}{12}$.
9. Reduce $156\frac{3}{5}$ to an improper fraction. *Ans.* $\frac{823}{5}$.
10. Change $20\frac{18}{68}$ to a fraction? *Ans.* $\frac{1378}{68}$.

SECTION IV.

1. How much is twice $\frac{1}{2}$ of an apple?
2. How much is 3 times $\frac{1}{8}$ of an apple?
3. How much is 3 times $\frac{1}{4}$ of an apple? 3 times $\frac{1}{7}$?
4. How much is 3 times $\frac{2}{11}$? 4 times $\frac{3}{13}$?
5. How much is 5 times $\frac{2}{13}$ of a foot?

If a foot is divided into 12 *parts, each part is* 1 *inch; and* 5 *times* $\frac{1}{12}$ *of a foot is* $\frac{5}{12}$ *of a foot, or* 10 *inches. Ans.*

How much is $\frac{5}{12}$ of 5 feet?

5 *feet*$=5\times12=60$ *inches, or* $\frac{60}{12}$; $\frac{1}{12}$ *of* 60 *inches is* 5 *inches; and* $\frac{5}{12}$ *of* 60 *inches is* 10 *inches, or* $\frac{10}{12}$ *of a foot.* Hence, 5 *times* $\frac{5}{12}$ *is the same as* $\frac{5}{12}$ *of* $5=\frac{10}{12}$.

6. How much is 24 times $\frac{3}{4}$, or $\frac{3}{4}$ of 24?

24 times $\frac{3}{4}=\frac{72}{4}=18$, and $\frac{3}{4}$ of $24=3$ times $6=18$; since 6 is $\frac{1}{4}$ of 24.

7. How much is 4 times $\frac{5}{24}$? *Ans.* $\frac{20}{24}$, or $\frac{5}{6}$.

4 times $\frac{1}{24}$ is $\frac{4}{24}$, or $\frac{1}{6}$, and,

4 times $\frac{5}{24}$ is $\frac{5}{6}=5$ times $\frac{1}{6}$,

found by dividing the denominator 24, by the multiplier 6.

8. How much is 3 times $\frac{5}{18}$? *Ans.* $\frac{5}{6}$. 3 times $\frac{5}{21}$? *Ans.* $\frac{5}{7}$.

9. How much is 7 times $\frac{3}{56}$? 5 times $\frac{3}{40}$?

10. What part of a whole number is 11 times $\frac{5}{77}$? 9 times $\frac{4}{63}$?

Hence, to multiply a fraction and a whole number together—

I.—*Divide the denominator of the fraction by the whole number, if it can be done without a remainder, and place the quotient under the numerator* (Ex. 7.) Otherwise,

II.—*Multiply the numerator by the whole number, and place the product over the denominator* (Ex. 1, 2, 4). Or,

III.—*Divide the whole number by the denominator, if it can be done without a remainder, and multiply the quotient by the numerator; the result (in this case) is a whole number* (Ex. 6).

If the numbers that compose the numerator and de-

nominator contains a common factor, cancel it by division; as,

11. Multiply $\frac{5}{48}$ by 36. *Thus:* $\frac{5\times\overset{3}{\cancel{36}}}{\underset{4}{\cancel{48}}}=\frac{5\times 3}{4}=\frac{15}{4}$ *by canceling* 12.

12. Multiply $\frac{7}{16}$ by 8; $\frac{5}{16}$ by 7. *Ans.* $\frac{7}{2}$, or $3\frac{1}{2}$.

13. What part of 1 is 7 times $\frac{5}{48}$? 5 times $\frac{7}{48}$?

14. What part of a bushel is 6 times $\frac{3}{7}$ of a bushel? Is it a proper or an improper fraction?

15. What part of a pound is $\frac{3}{14}$ of 21 pounds?

16. How many yards are there in $\frac{4}{5}$ of 12 yards?

17. How much is $\frac{11}{120}$ of 40? $\frac{11}{120}$ of 60?

18. How much is $\frac{2}{3}$ of 369? $\frac{25}{32}$ of 2? $\frac{1}{3}$ of 2 times 369?

19. What is 12 times $\frac{33}{4}$? 12 times $\frac{1}{2}$ of 36?

20. A person owning $\frac{1}{4}$ of a lottery ticket, will receive how much, if the ticket draws $7? If the ticket draws $11? $16?

21. A lady, meeting a beggar, gave her $\frac{3}{8}$ of what money she had: how much was that, if the lady had $10? If she had $7?

22. At 36 cents a bushel, what cost $\frac{1}{9}$ of a bushel of oats? What cost $\frac{3}{8}$ of a bushel? What is 36 times $\frac{3}{8}$? 9 times $\frac{3}{4}$?

23. At 44 cents a gallon for molasses, what will $\frac{1}{11}$ of a gallon cost? $\frac{3}{16}$ of a gallon? How much is $44\times\frac{3}{16}$? $11\times\frac{3}{4}$? *Ans.* $\frac{33}{4}=8\frac{1}{4}$ cents.

24. At 42 cents a pound for cinnamon, what cost $\frac{2}{7}$ of a pound? $\frac{3}{8}$ of a pound? $\frac{5}{8}$ of a pound?

How much is $42\times\frac{3}{8}$? $21\times\frac{5}{4}$?

25. At $\$\frac{3}{16}$ a yard for silk, how much will 3 yards cost? 6 yards? 11 yards? 12 yards?

26. If a man earn $\$\frac{7}{8}$ per day, how much will he earn in 3 days? in 5 days? 6 days?

SECTION V.

1. If lemons are $2\frac{1}{2}$ cents apiece, how much will 5 lemons cost?

Solution.—5 times $2\frac{1}{2}$ cents is 5 times $\frac{1}{2}$ cent and 5 times 2 cents. 5 times $\frac{1}{2}$ cent is $\frac{5}{2}$, or $2\frac{1}{2}$ cents; and 5 times 2 cents is 10 cents, to which add $2\frac{1}{2}$, making $12\frac{1}{2}$ cents. *Ans.* Or thus:

$2\frac{1}{2}=\frac{5}{2}$ cents=price of 1 lemon; 5 times $\frac{5}{2}=\frac{25}{2}=12\frac{1}{2}$.

Hence, to multiply a mixed number and a whole number together. Multiply the fraction, and then the whole number, and add the products together, or first reduce the mixed number to an improper fraction. The first method is best for large numbers.

2. Selling hats at $3\frac{3}{8}$ dollars each, how many dollars will be received for 4 hats? $\frac{3}{8}\times 4=\frac{3}{2}=1\frac{1}{2}$.

Ans. $\$13\frac{1}{2}$.

3. At $3\frac{3}{8}$ apiece, what would 5 hats cost? 11 hats? *Ans.* to the last, $\$37\frac{1}{8}$.

4. If a canal boat moves $2\frac{3}{4}$ miles an hour, how far will it move in 12 hours? in 17 hours?

Ans. $46\frac{3}{4}$ miles.

5. How many days are there in $\frac{7}{15}$ of a month of 30 days?

6. How many quarts are there in $4\frac{3}{8}$ gallons? 4 quarts=1 gallon.

7. If a man sows $2\frac{2}{5}$ bushels of oats on an acre, how many bushels will he sow on 10 acres? 11 acres?

8. At $\$29$ for 26 yards of cloth, how much will 104 yards cost?

9. If a man travels $2\frac{8}{40}$ miles an hour, how far will he go in 5 hours? 8 hours? 20 hours?

10. What is 5 times $6\frac{1}{3}$? 5 times $6\frac{1}{8}$?

11. What is 4 times $12\frac{1}{8}$? 4 times $12\frac{1}{4}$?

12. How many yards in 6 times $5\frac{1}{2}$ yards?

13. How many bushels in 8 times $7\frac{3}{4}$ bushels?

14. How many feet of timber are there in 7 sticks containing $12\frac{1}{8}$ feet each?

15. How many acres are in 4 fields, each containing $10\frac{1}{8}$ acres?

SECTION VI.

1. A man owning $\frac{1}{7}$ of a mill, sold $\frac{1}{4}$ of his share? what part of the mill did he sell?

Solution.—$7 \times 4 = 28$, the product of the two denominators. If the mill were divided into 28 parts, (7 times 4,) then $\frac{1}{7}$ of the whole would be $28 \div 7 = 4$ of those parts, that is, $\frac{1}{7} = \frac{4}{28}$ which the man owned, and $\frac{1}{4}$ of those 4 parts is 1 part $= \frac{1}{28}$, which he sold. *Ans.* $\frac{1}{28}$. Hence $\frac{1}{4}$ of $\frac{1}{7} = \frac{1}{28}$, the numerator being 1, or once 1 and the denominator 4 times 7.

2. A boy having $\frac{1}{4}$ of a dollar, gave $\frac{1}{3}$ of his money for a knife, what part of a dollar did the knife cost him?

3. What part of an orange is $\frac{1}{5}$ of $\frac{1}{6}$ of an orange?

4. How much is $\frac{1}{8}$ of $\frac{1}{4}$ of a foot? What part of a foot? How many inches?

5. How much is $\frac{1}{8}$ of $\frac{1}{3}$? $\frac{1}{4}$ of $\frac{1}{5}$? $\frac{1}{7}$ of $\frac{1}{12}$?

6. How much is $\frac{1}{2}$ of $\frac{1}{2}$? $\frac{1}{3}$ of $\frac{1}{3}$? $\frac{1}{5}$ of $\frac{1}{5}$? $\frac{1}{10}$ of $\frac{1}{10}$?

7. How much is $\frac{1}{8}$ of $\frac{1}{16}$? $\frac{1}{17}$ of $\frac{1}{18}$? $\frac{1}{18}$ of $\frac{1}{18}$?

8. How much is $\frac{1}{217}$ of $\frac{1}{218}$? $\frac{1}{218}$ of $\frac{1}{219}$?

9. How much is $\frac{1}{1000}$ of $\frac{1}{10}$? $\frac{1}{350}$ of $\frac{1}{370}$?

10. How much is $\frac{1}{2500}$ of $\frac{1}{1000}$? $\frac{1}{370}$ of $\frac{1}{370}$?

SECTION VII.

1 A man owning $\frac{5}{7}$ of a mill, sold $\frac{1}{4}$ of his share; what part of the mill did he sell?

Solution.—By the last section, when the man owned $\frac{1}{7}$ and sold $\frac{1}{4}$ of his share, he sold $\frac{1}{28}$ of the whole. But $\frac{1}{4}$ of $\frac{5}{7}$ must be 5 times as much as $\frac{1}{4}$ of $\frac{1}{7}$, Therefore it is 5 times $\frac{1}{28}$, which is $\frac{5}{28}$. That is, $\frac{1}{4}$ of $\frac{5}{7}=\frac{5}{28}$, or $\frac{5\times1}{7\times4}$; in which the two numerators are multiplied together, and the two denominators are multiplied together.

2. How much is $\frac{5}{7}$ of $\frac{1}{4}$? $\frac{5}{4}$ of $\frac{1}{7}$?

3. How much is $\frac{5}{7}$ of $\frac{1}{2}$? $\frac{5}{7}$ of $\frac{2}{3}$?

4. What part of a foot is $\frac{2}{4}$ of $\frac{1}{3}$ of a foot? How many inches?

5. $\frac{1}{3}$ of the stock on a certain farm consisted of cows, and the owner sold $\frac{1}{5}$ of them; if the whole number of animals was 15, how many cows did he sell? How many, if the whole number was 30? How many if the whole number was 45? Could the whole number have been any less than 15, or any number between 15 and 30; or between 30 and 45?

6. What part of 100, is $\frac{1}{4}$ of $\frac{2}{15}$ of 100?

7. What part of 1 is $\frac{1}{4}$ of $\frac{2}{15}$ of 1?

8. What part of 3 is $\frac{1}{4}$ of $\frac{2}{15}$ of 3?

SECTION VIII.

1. A man owning $\frac{5}{7}$ of a mill, sold $\frac{3}{4}$ of his share; what part of the mill did he sell?

Solution.—When the man owned $\frac{5}{7}$, and sold $\frac{1}{4}$ of his share, he sold $\frac{5}{28}$ of the whole; but if he sold $\frac{3}{4}$ of his

share, he sold 3 times $\frac{5}{28}$, which is $\frac{15}{28}$ of the whole. Then, $\frac{3}{4}$ of $\frac{5}{7}=\frac{3\times5}{4\times7}$, $\frac{15}{28}$. Therefore,

To find the value of a fraction of a fraction, *multiply the numerators together for a new numerator, and the denominators together for a new denominator.*

2. How much is $\frac{3}{5}$ of $\frac{5}{7}$? $\frac{5}{7}$ of $\frac{3}{5}$? $\frac{3}{5}$ of $\frac{5}{8}$? $\frac{5}{8}$ of $\frac{3}{7}$?
3. How much is $\frac{4}{5}$ of $\frac{3}{8}$? $\frac{5}{11}$ of $\frac{7}{12}$? $\frac{6}{7}$ of $\frac{5}{11}$?
4. How much is $\frac{5}{9}$ of $\frac{7}{9}$? $\frac{6}{5}$ of $\frac{8}{11}$?
5. How much is $\frac{2}{3}$ of $\frac{3}{5}$ of $\frac{3}{4}$?

Solution.—$\frac{3}{5}$ of $\frac{3}{4}=\frac{9}{20}$; and $\frac{2}{3}$ of $\frac{9}{20}=\frac{18}{60}=\frac{3}{10}$. *Ans.* Therefore $\frac{2}{3}$ of $\frac{3}{5}$ of $\frac{3}{4}=\frac{2\times3\times3}{3\times5\times4}=\frac{18}{60}$; but if we cancel 2 and 3, the result is $\frac{3}{10}$, in its lowest terms.

6. When apples are $\$\frac{1}{4}$ a bushel, what will $\frac{3}{5}$ of a bushel cost?

$\frac{3}{5}$ of a bushel will cost $\frac{3}{5}$ times the price, or $\frac{3}{5}$ of the price of 1 bushel? *Ans.* $\$\frac{3}{20}$.

This last answer $\$\frac{3}{20}=\$\frac{15}{100}=15$ cents. The price, $\$\frac{1}{4}=25$ cents,—and $\frac{3}{5}$ of 25 cents$=15$ cents.

A *simple fraction* has one numerator and one denominator; as, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$, $\frac{8}{9}$.

A *compound fraction* is a *fraction of a fraction*, or several fractions connected by the word *of;* as, $\frac{3}{5}$ of $\frac{5}{7}$; $\frac{1}{2}$ of $\frac{3}{5}$ of $\frac{5}{6}$.

The foregoing examples show that,

A compound fraction is the same as the *product of two or more fractions;* that is, the word *of*, is the *sign of multiplication.*

The same is true in relation to whole numbers; as, tens of tens are hundreds.

Hence, To multiply fractions together, or to reduce a compound fraction to a simple one.

Multiply the numerators together for a new numera-

tor, and the denominators together for a new denominator.

Mixed numbers must be prepared by reducing them to improper fractions.

NOTE.—In applying this rule, there is great advantage in cancelling common factors.

7. Multiply $\frac{7}{8}$ by $\frac{8}{7}$. *Ans.* $\frac{8}{8}$, by cancelling the 7s.

8. How much is $\frac{1}{3}$ of $\frac{3}{4}$ of $\frac{2}{4}$ of $\frac{4}{5}$?

9. Multiply together 7, $\frac{8}{9}$, $\frac{3}{7}$, and $\frac{6}{11}$ of $\frac{11}{12}$.

Ans. 1.

Thus: $\frac{\cancel{7}\ \cancel{8}\ \cancel{3}\ \cancel{6}\ \cancel{11}}{1\ \cancel{9}\ \cancel{7}\ \cancel{11}\ \cancel{12}}=1.$

10. Multiply $2\frac{1}{4}$ by $3\frac{1}{2}$.

$2\frac{1}{4}=\frac{9}{4}$; $3\frac{1}{2}=\frac{7}{2}$; $\frac{9}{4}\times\frac{7}{2}=\frac{63}{8}=7\frac{7}{8}$. *Ans.*

Reduce mixed numbers to improper fractions.

11. Multiply $8\frac{3}{4}$ by $\frac{8}{7}$, (Cancel 7.) *Ans.* $3\frac{3}{4}$.

12. Multiply $1\frac{7}{8}$ by $2\frac{1}{2}$, *Ans.* $3\frac{3}{16}$.

Multiply $\frac{1}{2}$ of 3 by $\frac{1}{3}$ of 6. $\frac{1}{3}$ of 4 by $\frac{1}{4}$ of 5.

Ans. $1\frac{1}{2}$.

Thus: $\frac{1\ \ 4\ \ 1\ \ 5}{\cancel{3}\ \ \cancel{1}\ \ \cancel{6}\ \ \cancel{1}}=\frac{10}{6}$, by canceling 2.

13. Multiply $\frac{7}{8}$ of $4\frac{1}{2}$ by $\frac{1}{3}$ of $3\frac{1}{2}$. *Ans.* $1\frac{1}{4}$.

14. Find the continued product of $\frac{3}{4}$ of $\frac{2}{3}$, 7, $5\frac{1}{2}$, and $\frac{1}{3}$ of $\frac{4}{5}$. *Ans.* $4\frac{1}{10}$.

15. Multiply together $2\frac{1}{2}$, $6\frac{2}{3}$, $3\frac{1}{4}$.

SECTION IX.

1. If 2 yards of calico cost $\$\frac{1}{2}$, what cost 1 yard?

Solution—1 yard will cost 1 half as much as 2 yards; $\frac{1}{2}$ of $\$\frac{1}{2}$ is $\$\frac{1}{4}$. *Ans.*; by dividing the numerator of $\frac{1}{2}$ by 2. This is dividing the fraction $\frac{1}{2}$ by 2.

2. If 3 bushels of apples cost $\$\frac{4}{7}$, what will 1 bushel cost?

3. If 4 yards of cloth cost $\$\frac{1}{11}$, what will 1 yard cost?

4. At $6 a barrel for flour, what part of a barrel will $\$2\frac{2}{3}$ buy?

Reduce the dividend to an improper fraction. How many times, or what part of a time, is the price, $6, contained in the cost, $\$\frac{12}{5}$? *Ans.* $\frac{2}{5}$.

5. If 7 bushels of wheat cost $\$8\frac{3}{4}$, what will 1 bushel cost? *Ans.* $\$1\frac{1}{4}$.

6. If 7 yards of cloth cost $\$30\frac{1}{8}$, what will 1 yard cost?

When the dividend is greater than the divisor, divide the whole number of the dividend first; then divide the remainder; thus, 7 in $30\frac{1}{8}$, 4 times and $2\frac{1}{8}$ remains; $2\frac{1}{8}=\frac{17}{8}$; $\frac{17}{8}\div 7=\frac{1}{8}$. *Ans.* $4\frac{1}{8}$.

7. $42\frac{2}{7}$ is how many times 8? how many times 4? How many times 16? How many times 32?

8. $17\frac{1}{7}$ is how many times 2? how many times 3? 4? 5? *Answers,* $8\frac{1}{7}$; $5\frac{5}{7}$; $4\frac{2}{7}$; $3\frac{3}{7}$.

9. $10\frac{10}{11}$ is how many times 2? 3? 4? 5? *Answers,* $5\frac{8}{11}$; $3\frac{7}{11}$; $2\frac{8}{11}$; $2\frac{0}{11}$?

SECTION X.

1. If 5 yards of calico cost $\$\frac{1}{4}$, what will 1 yard cost?

$\$\frac{1}{4}=\$\frac{5}{20}$; and $\frac{1}{5}$ of $\frac{5}{20}$ is $\frac{1}{20}$. *Ans.* $\$\frac{1}{20}$.

Hence we have two ways to divide a fraction by a whole number.

I.—*Divide the numerator by the whole number when it can be done without a remainder, and under the quotient place the denominator; otherwise,*

II.—*Multiply the denominator by the whole number, and over the product place the numerator.*

If the divisor and the numerator contain a common factor, cancel that factor; thus,

2. Divide $\frac{8}{9}$ by 16. 8 is a factor of 8 and 16.
Then $\frac{8}{9} \div 16 = \frac{1}{9} \div 2 = \frac{1}{18}$. *Ans.*

3. Divide $\frac{3}{4}$ by 4. *Ans.* $\frac{3}{16}$.

4. Divide $\frac{3}{27}$ by 5; by 20; by 30; by 40; by 64; by 65. *Answers.* $\frac{4}{27}$, $\frac{1}{27}$, $\frac{2}{81}$, $\frac{1}{54}$, $\frac{5}{432}$, $\frac{4}{351}$.

5. If 8 lbs. of coffee cost $\frac{2}{25}$ of a dollar, what is the price per lb.? *Ans.* $\frac{2}{25}$ of a dollar.

6. If 10 lbs. of lead cost $\frac{4}{5}$ of a dollar, what is the price per lb.? *Ans.* $\frac{8}{25}$ of a dollar.

7. If $\frac{35}{36}$ of an acre produce 29 bushels of wheat, what part of an acre will produce 1 bushel?
Ans. $\frac{36}{1044}$ of an acre.

8. If $\frac{35}{36}$ of an acre produce 28 bushels, how much land will produce 1 bushel?
Ans. $\frac{5}{144}$ of an acre.

9. Divide $5\frac{7}{8}$ by 11. *Ans.* $\frac{47}{88}$.
Thus: $5\frac{7}{8} = \frac{47}{8}$: $\frac{47}{8} \div 11 = \frac{47}{88}$.

10. How many times 7 is $\frac{21}{25}$? or rather, since 7 is greater than $\frac{21}{25}$, what part of 7 is $\frac{21}{25}$? *Ans.* $\frac{3}{25}$.

11. How many times 11 is $8\frac{1}{6}$?

12. What part of 13 is contained in $7\frac{4}{5}$?

13. What part of 8 is $9\frac{1}{8}$? $1\frac{1}{8}$?

14. What part of 21 is $7\frac{7}{11}$?

15. How many times 28 is $16\frac{4}{7}$?

16. How many times 16 is $33\frac{3}{5}$?

17. How many times 11 is $17\frac{2}{7}$?

18. What part of 3 is $5\frac{1}{3}$? $2\frac{2}{3}$?

19. $577\frac{1}{2}$ is how many times 3? 4? 5? 7? 11?
Answers. $192\frac{1}{2}$; $144\frac{3}{8}$; $115\frac{1}{2}$; $82\frac{1}{2}$; $52\frac{1}{2}$?

20. If 7 yards of cloth cost $\$\frac{9}{10}$, how many cents will 1 yard cost? *Ans.* $12\frac{6}{7}$ cents.

21. If 9 bushels of wheat are worth $30\frac{3}{4}$ bushels of oats, how many bushels of oats are worth 1 bushel of wheat?

22. If a steamboat goes 12 miles while a train of cars goes $25\frac{1}{2}$, how far does the train go while the steamboat goes 1 mile? *Ans.* $2\frac{1}{8}$ miles.

23. If $25\frac{1}{3}$ tons of hay grow on 16 acres, how much hay grows on 1 acre? *Ans.* $1\frac{7}{12}$ tons.

24. If $\$165\frac{3}{4}$ are paid for 25 barrels of flour, what is the price per barrel? *Ans.* $\$6\frac{7}{100}$, or $\$6.62\frac{3}{4}$.

25. If $\$3\frac{4}{7}$ are paid for 75 oranges, what part of a dollar is the price of 1 orange? *Ans.* $\frac{1}{21}$.

26. If a man pays a tax of $\$5\frac{1}{2}$ on \$500, how much is such tax on 1 dollar? *Ans.* $\$\frac{11}{1000}$.

SECTION XI.

1. How many times can a measure containing $\frac{3}{4}$ of a gallon, be filled from 5 gallons of oil?

Solution.—As many times as the fraction $\frac{3}{4}$ is contained in 5. Reducing the 5 gallons to fourths, they make 5 times 4, 20 fourths, 3 fourths are contained in 20 fourths, as many times as 3 gallons are contained in 20 gallons, which is $6\frac{2}{3}$ times. *Ans.* Therefore, $5 \div \frac{3}{4}$ is $6\frac{2}{3}$.

2. If one load of wood is $\frac{3}{4}$ of a cord, what part of a load is 1 cord?

1 cord $=\frac{4}{4}$, and $\frac{4}{4} \div \frac{3}{4} = 4 \div 3 = \frac{4}{3}$. *Ans.*
Therefore $1 \div \frac{3}{4} = \frac{4}{3}$.

3. How much is $1 \div \frac{1}{3}$? $1 = \frac{3}{3}$; $\frac{3}{3} \div \frac{1}{3} = 3 \div 4 = \frac{3}{4}$.

4. How much is $1 \div \frac{5}{7}$? $1 \div \frac{7}{7}$?

5. How much is $1 \div 1\frac{2}{5}$? $1 \div 1\frac{5}{3}$.

6. If $\frac{3}{4}$ is contained in 1, $\frac{4}{5}$ times, how many times is it contained in 5?

Solution.—*5 times as many times as it is in* 1. *5 times* $\frac{4}{5} = \frac{20}{5} = 6\frac{2}{3}$, *which agrees with question* 1.

7. How many times is $\frac{3}{4}$ contained in $\frac{5}{7}$?

Solution.—$\frac{3}{4}$ is contained in $\frac{5}{7}$, $\frac{5}{7}$ as many times as $\frac{3}{4}$ is contained in 1: that is $\frac{5}{7}$ of $\frac{4}{3}$ times $= \frac{20}{21}$. Therefore $\frac{5}{7} \div \frac{3}{4} = \frac{5}{7} \times \frac{4}{3}$.

When the denominator of a fraction is written over the numerator the fraction is *inverted*. Hence, to divide by a fraction,

Invert the divisor, and proceed as in multiplication.

This rule will apply also to a whole number; thus,

8. Divide $\frac{7}{8}$ by 4. $4 = \frac{4}{1}$; and $1 \div 4 = \frac{1}{4}$.

Therefore $\frac{7}{8} \div 4 = \frac{7}{8} \times \frac{1}{4} = \frac{7}{32}$. *Ans.*

9. Divide $8\frac{1}{4}$ by $6\frac{1}{2}$. $8\frac{1}{4} = \frac{33}{4}$; $6\frac{1}{2} = \frac{13}{2}$. $\frac{33}{4} \div \frac{13}{2} \div \frac{33}{4} \times \frac{2}{13} = \frac{33}{26} = 1\frac{7}{26}$, by cancelling 2.

Reduce mixed numbers to improper fractions, and cancel as in multiplication of fractions.

10. $15\frac{1}{2}$ is how many times $3\frac{1}{2}$?

Ans. $\frac{13}{3}$ or $4\frac{1}{3}$ times.

11. $3\frac{1}{2}$ is how many times $15\frac{1}{6}$? *Ans.* $\frac{3}{13}$ times.

12. $\frac{2}{3}$ of $\frac{3}{4}$ of 6 is how many times $\frac{1}{3}$ of 5? or, is what part of $\frac{1}{3}$ of 5?

The questions—*how many times*, and, *what part of*, both mean the same; except that the first form seems to imply more than 1, and the second less, but the solution must prove which is correct in each example.

$\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{6}{1} \times \frac{3}{1}$ of $\frac{1}{5} = \frac{9}{2}$. *Ans.* $1\frac{4}{2}$.

Invert every factor of the divisor.

Proof. $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{6}{1} = \frac{9}{2}$ of $\frac{1}{5}$ of $\frac{5}{1}$; for each is equal to 3.

13. How many times $\frac{2}{3}$ is $\frac{3}{4}$? How many times $\frac{3}{4}$ is $\frac{2}{3}$?

14. Divide $\frac{3}{5}$ by $\frac{6}{7}$. $\frac{6}{7}$ by $\frac{3}{5}$.

15. How many times $\frac{2}{3}$ is $2\frac{1}{3}$?

$2\frac{1}{3}$ is how many times $3\frac{1}{2}$?

16. How many times $\frac{3}{4}$ is $2\frac{1}{6}$?

$2\frac{1}{6}$ is how many times $1\frac{5}{8}$?

17. How many times $\frac{3}{4}$ of $\frac{4}{7}$ is $\frac{9}{10}$?

$\frac{9}{10}$ is how many times $2\frac{1}{10}$?

18. If $\$\frac{5}{6}$ buy a bushel of rye, what part of a bushel will $\$\frac{3}{4}$ buy? If the question were how many bushels will $\$\frac{3}{4}$ buy, the pupil would perceive that he must see how many times $\frac{5}{6}$ is in $\frac{3}{4}$, and that is the question now. *Ans.* $\frac{9}{10}$ bushel.

If $\$\frac{3}{4}$ will buy $\frac{9}{10}$ of a bushel of rye; what is the price of 1 bushel? *Ans.* $\$\frac{5}{6}$.

19. At $\$\frac{3}{5}$ per yard, how much cloth can you buy for $\$2\frac{2}{3}$.

20. If $4\frac{1}{2}$ yards of cloth cost $\$2\frac{2}{3}$ what is the price per yard?

21. $\frac{3}{7}$ is $\frac{4}{5}$ of what number? $\frac{3}{7}$ is $\frac{15}{28}$ of what number?

22. $\frac{5}{8}$ is $\frac{5}{7}$ of $\frac{1}{2}$ of what number?

$\frac{5}{8}$ is $1\frac{5}{4}$ times $\frac{1}{2}$ of what number?

23. 7 is $\frac{3}{5}$ of what number?

SECTION XII.

1. $1\frac{1}{3}$ is twice what number?
2. 2 is 3 times what number?
3. $2\frac{2}{3}$ is 4 times what number?
4. 3 is $4\frac{1}{2}$ times what number?

5. $3\frac{1}{2}$ is $5\frac{1}{4}$ times what number?

Answer, to the five: $\frac{3}{5}$.

Solution to the last, $3\frac{1}{2}=\frac{7}{2}$; $5\frac{1}{4}=\frac{21}{4}$.

$\frac{7}{2}$ is $\frac{21}{4}$ times a number?

What is 1 time, once that number?

As many times as $\frac{21}{4}$ must be taken to make 1, so many times $\frac{7}{2}$ must be taken to make that number.

$\frac{21}{4}$ in 1, $\frac{4}{21}$ times, $\frac{4}{28}$ times $\frac{7}{2}$, or $\frac{4}{21}$ of $\frac{7}{2}=\frac{2}{3}$. *Ans.*

6. $\frac{3}{5}$ is once what number?

7. $1\frac{1}{3}$ is twice what number?

8. $1\frac{1}{2}$ is $2\frac{1}{2}$ times what number?

9. $1\frac{1}{3}$ is 3 times what number?

10. 2 is $3\frac{1}{3}$ times what number?

11. $2\frac{2}{3}$ is 4 times what number?

12. $2\frac{2}{5}$ is $4\frac{1}{3}$ times what number?

Same answer to 6, 7, 8, 9, 10, 11, 12.

13. How many times $\frac{3}{4}$ is $\frac{5}{7}$?

$\frac{5}{7}$ is how many times in 1? $\frac{7}{5}$ times. In $\frac{3}{4}$? $\frac{3}{4}$ of $\frac{7}{5}$ times. *Ans.* $1\frac{1}{20}$.

14. How many times $\frac{2}{5}$ is $\frac{1}{10}$?

15. How many times $\frac{2}{7}$ is $\frac{6}{11}$?

16. How many times $\frac{6}{10}$ is $\frac{3}{7}$?

17. How many times $\frac{5}{6}$ is $\frac{7}{12}$?

18. How many times $\frac{2}{4}$ is $2\frac{2}{3}$?

19. How many times $1\frac{1}{2}$ is $2\frac{1}{3}$?

20. How many times $1\frac{2}{5}$ is $2\frac{1}{5}$?

21. How many times $2\frac{2}{4}$ is $3\frac{1}{6}$?

10. A obtained 48 bushels of corn from an acre of ground, which was but $\frac{4}{5}$ as much as B obtained; how much did B obtain?

11. B obtained 60 bushels of corn from an acre; A obtained but $\frac{4}{5}$ as much: how much did A obtain?

12. If by success in trade, 1 dollar becomes $\$1\frac{1}{2}$, what sum would become \$100?

13. If 5 yards of cloth cost $7, how much cloth can you buy for $1?

14. If 100 pounds of flour cost $3\frac{1}{2}$, how much flour can you buy for $1?

15. Paying $\$\frac{5}{8}$ per bushel for potatoes, how many bushels can you buy for $1?

16. James being asked his age, said that $8\frac{1}{2}$ years was $\frac{3}{4}$ of $\frac{3}{5}$ of his age; how old was he?

Ans. $21\frac{1}{4}$ years.

SECTION XIII.

Reasoning by equations or equalities, is more extensive in its applications than any other method. It depends on the principle that if equal quantities are equally increased or diminished, the results are equal.

1. If $7\frac{1}{2}$ yards of cloth cost $\$8\frac{1}{3}$, what is the price per yard?

Cost of $\frac{15}{2}$ yards $=\frac{25}{2}$ dollars.
Price of 1 yard = how many dollars?

Now, what multiplier, multiplied into $\frac{15}{2}$ yards, will make 1 yard? $\frac{2}{15}$. Then the same multiplier into $\frac{25}{3}$ dollars will give the answer, to put in the place of the question how many dollars? $\$\frac{25}{3}\times\frac{2}{15}=\$\frac{50}{45}=\$1\frac{5}{9}$. *Ans.*

2. If James earns $\$32\frac{1}{2}$ while Peter earns $13, how much will Peter earn while James earns $1?

Time of J. earning $\frac{65}{2}$ = time of P. earning 13.
Time of J. earning 1 = time of P. earning how much?
Time of J. earning 1 = time of P. earning $13\times\frac{2}{65}$.
Ans. $\$\frac{2}{5}$.

3. If $59\frac{1}{2}$ yards of cloth cost $\$29\frac{3}{4}$, how many yards will $\$31\frac{1}{4}$ buy?

Cost of $1\frac{1}{2}$ yards $=1\frac{1}{4}$ dollars.
Cost of how many yards $=1\frac{2}{4}$ dollars.
As many times as $\$1\frac{1}{4}$ is contained in $\$1\frac{2}{4}$, so many times is $1\frac{1}{2}$ contained in the required number of yards.
Therefore $1\frac{1}{2} \times 1\frac{2}{4} \times \frac{4}{1\,1} = 1\frac{2}{2} = 62\frac{1}{2}$ yards. *Ans.*

SECTION XIV.

A fraction may be charged to an equivalent fraction having a given denominator by multiplying both terms by the number of times that its denominator is contained in the new denominator; as,

1. $\frac{1}{2}$ is how many 4ths? 2 in 4, 2 times.
Therefore $\frac{1}{2} = \frac{2}{4}$, by multiplying both terms by 2.
2. $\frac{3}{4}$ = how many 8ths? 12ths? 20ths?
Ans. $\frac{3}{4} = \frac{6}{8}$ of $\frac{3}{4} = \frac{9}{12} = \frac{3}{4}$ of $\frac{3}{4} = \frac{15}{20}$.
3. $\frac{5}{8}$ = how many 40ths? 24ths? 32ds?
4. In $\frac{4}{7}$ are how many 14ths? 28ths? 35ths?
5. In $\frac{3}{8}$ is there a whole number of 12ths? 14ths? 16ths? 18ths? 24ths? 28ths? 30ths? 40ths?
6. Change $\frac{2}{3}$ and $\frac{3}{5}$ to the same denominator $(3 \times 5 =)$ 15.
$\frac{2}{3} = \frac{10}{15}$; $\frac{3}{5} = \frac{9}{15}$. *Ans.* $\frac{10}{15}$ and $\frac{9}{15}$.

The same denominator belonging to different fractions is called a *common denominator*. To find a common denominator, *seek for the least number that contains each of the different denominators without a remainder.*

7. Reduce $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$, to a common denominator.
Ans. $\frac{6}{12}$, $\frac{8}{12}$, $\frac{9}{12}$.
8. Reduce $\frac{3}{8}$, $\frac{1}{4}$, and $\frac{5}{16}$ to a c. d. *Ans.* $\frac{6}{16}$, $\frac{4}{16}$, $\frac{5}{16}$.
9. In $\frac{5}{12}$ and $\frac{3}{8}$ are how many 24ths?
$\frac{5}{12} = \frac{10}{24}$; $\frac{3}{8} = \frac{9}{24}$. $\frac{10+9}{24} = \frac{19}{24}$, *Ans.* $\frac{19}{24}$.

The reducing of fractions to a common denominator, is to prepare them for addition and subtraction; for then only are the numerators fractional units of the same magnitude.

SECTION XV.

1. Peter has $\$\frac{1}{8}$, and James $\$\frac{5}{8}$; how much money have they both?

2. How much is $\frac{3}{8}$ and $\frac{1}{4}$? The sum is neither $\frac{4}{8}$ nor $\frac{4}{4}$; but $\frac{1}{4}=\frac{2}{8}$, and $\frac{3}{8}+\frac{2}{8}=\frac{5}{8}$. *Ans.*

So also, $\frac{3}{8}-\frac{2}{8}=\frac{1}{8}$, the difference of the fractions.

Therefore, to add or subtract fractions,

Reduce to a common denominator; then add or subtract the numerators.

3. How many oranges are there in $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{8}$, and $\frac{7}{12}$ of an orange? *Ans.* 2.

4. How much is $\frac{7}{12}$ less than $\frac{2}{3}$? $\frac{7}{12}$ more than $\frac{1}{2}$?

5. If you should borrow $\$\frac{3}{4}$, and pay but $\frac{2}{5}$ of the debt, how much would remain unpaid?

Ans. $\$\frac{6}{20}$, or $\frac{3}{10}$.

6. A laborer worked $\frac{1}{2}$ of the forenoon, and $\frac{1}{3}$ of the afternoon; what part of the day did he work?

Ans. $\frac{5}{12}$.

7. What is the sum of $\frac{3}{4}$ and $\frac{5}{8}$? $\frac{3}{5}$ and $\frac{2}{7}$?

8. Find the sum and difference of $\frac{4}{7}$ and $\frac{2}{3}$.

9. What is the difference of $\frac{15}{16}$ and $\frac{3}{4}$?

10. What is the sum of $\frac{7}{9}$ and $\frac{2}{7}$? $\frac{7}{8}$ and $\frac{3}{5}$?

11. Add and subtract $\frac{6}{7}$ and $\frac{3}{4}$?

12. What is the sum of $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{7}{16}$?

13. What is the sum of $2\frac{3}{4}$ and $5\frac{3}{4}$?

$2\frac{3}{4}+5\frac{3}{4}=7+\frac{6}{4}=8\frac{1}{2}$. *Ans.*

Add the whole numbers, and the fractions, separately.

14. Add $2\frac{3}{4}$, $5\frac{1}{2}$, and $6\frac{3}{8}$? *Ans.* $14\frac{5}{8}$.

15. What is the difference of $5\frac{1}{3}$ and $2\frac{5}{9}$?

$$5\frac{3}{9}-2\frac{5}{9}=2\frac{7}{9}.$$

Since $\frac{5}{9}$ cannot be taken from $\frac{3}{9}$, add $\frac{9}{9}$ to the $\frac{3}{9}$, then take $\frac{5}{9}$ from the sum $\frac{12}{9}$, and carry 1 to 2; 3 from 5, 2.

16. How much is $\frac{5}{6}+\frac{2}{3}$? $\frac{5}{6}-\frac{2}{3}$.

17. How much is $\frac{3}{4}+\frac{7}{8}$? $\frac{7}{8}-\frac{3}{4}$?

18. How much is $\frac{2}{3}+\frac{3}{7}+\frac{5}{14}$?

19. How much is $\frac{3}{5}+\frac{7}{10}-\frac{2}{20}$?

20. How much is $5\frac{1}{2}+3\frac{3}{7}$?

21. Add $6\frac{3}{5}+5\frac{3}{6}$.

22. From $8\frac{3}{5}$ take $7\frac{3}{9}$.

23. From 1 take $\frac{5}{6}$. From 2 take $\frac{5}{6}$.

24. How much is $1-\frac{3}{5}$? $1-\frac{7}{10}$? $2-\frac{5}{7}$?

25. How much is $8-2\frac{1}{2}$? $5-1\frac{1}{3}$? $6-2\frac{1}{3}$?

26. How much is $12\frac{1}{2}-7\frac{1}{2}$? $12\frac{1}{3}-2\frac{1}{6}$?

27. $\frac{1}{2}$ of the students in a school study arithmetic; $\frac{1}{3}$ geography; and the rest, grammar; what part of them study grammar? If there are 20 that study grammar, how many students are in the school? 20 is $\frac{1}{6}$ of what number?

28. $\frac{3}{5}$ of a certain army was killed, $\frac{1}{9}$ fled, and the remainder, 2600 men, fled; how many men were in the army? *Ans.* 14400.

29. An orchard consists of $\frac{1}{3}$ of its number of peach trees, $\frac{1}{6}$ cherry trees, 57 plum trees, and 209 apple trees; how many trees in the orchard?

209+57 is $\frac{7}{9}$ of what number? *Ans.* 570.

30. A market woman sold $\frac{1}{2}$ of her oranges at 4 cents a piece; $\frac{3}{5}$ of them at 3 cents; and the remainder, consisting of 18 oranges, at 2 cents; how much did she receive for the whole? *Ans.* $4.86.

CHAPTER VII.

COMPOUND NUMBERS.

SECTION I.

Simple numbers express things of the *same denomination;* as, 3 men, 10 books.

Numbers which express *different denominations,* as, the divisions of money, weight, measure, &c., are *compound numbers;* as, 5 shillings, 6 pence; 7 days, 12 hours, &c.

TABLES OF WEIGHTS AND MEASURES.

FEDERAL MONEY. THE CURRENCY OF THE UNITED STATES.

10 mills, (*m.*)	make	1 cent,	*ct.*
10 cents . . .		1 dime,	*d.*
10 dimes . . .		1 dollar, *dall. or*	$.
10 dollars . . .		1 eagle,	*E.*

ENGLISH OR STERLING MONEY.

4 farthings, (*qr. or far.*),	make	1 penny,	*d.*
12 pence		1 shilling,	*s.*
20 shillings		1 pound,	£ *or l.*
21 shillings		1 guinea.	

TROY WEIGHT.

24 grains, (*gr.*)	make	1 pennyweight,	*pwt. or dwt.*
20 pennyweights	.	1 ounce,	*oz.*
12 ounces . . .		1 pound,	*lb.*

AVOIRDUPOIS WEIGHT.

16 drams, (*dr.*)	. make	1 ounce,	*oz.*
16 ounces		1 pound,	*lb.*
25 pounds		1 quarter,	*qr.*
4 quarters, or 100 lbs.		1 hundred weight,	*cwt.*
20 hundred weight, or 2000 pounds . . .		1 ton,	*T.*

APOTHECARIES' WEIGHT.

20 grains (*gr.*)	make	1 scruple. *sc. or*	℈.
3 scruples . . .		1 dram, *dr. or*	ʒ.
8 drams . . .		1 ounce, *oz. or*	℥.
12 ounces . . .		1 pound,	*lb.*

LONG MEASURE.

12	inches (*in.*) make	.	1 foot, . . .	*ft.*
3	feet		1 yard, . . .	*yd.*
5½	yards or 16½ feet	.	1 rod or pole	*r.* or *p.*
40	rods.		1 furlong, .	*fur.*
8	furlongs or 320 rods		1 mile, . . .	*m.*
3	miles		1 league, . .	*l.*
69	geographical miles or 69½ statute miles,		1 degree, *deg.* or °	
360	degrees		the circumference of the earth.	

CLOTH MEASURE.

2¼	inches, (*in.*)	make 1	nail.	*na.*
4	nails	1	quarter of a yard,	*yr.*
4	quarters . . .	1	yard,	*yd.*
3	quarters . . .	1	Flemish Ell,	*Fl. E.*
5	quarters . . .	1	English Ell,	*E. E.*
6	quarters . . .	1	French Ell,	*Fr. E.*

SQUARE MEASURE.

144	sq. inches, (*sq. in.*) make	1	sq. foot, *sq. ft.*
9	sq. feet.	1	sq. yard, *sq. yd.*
30¼	sq. yards.	1	sq. pole, *sq. r. or p.*
272¼	sq. feet	1	sq. rod or pole.
40	sq. rods or poles . .	1	rood. *R.*
4	roods	1	acre, *A.*
640	acres.	1	sq. mile, *S. M.*

CUBIC MEASURE.

1728	cubic inches (*cu. in.*) make	1	cubic foot, *cu. ft.*
27	cubic feet	1	cubic yard, *cu. yd.*
40	feet	1	ton *T.*
16	cubic feet	1	cord foot, *c. ft.*
8 cord feet or 128	cubic feet	1	cord, *C.*

BEER MEASURE.

2	pints, (*pts.*) make	1	quart, *qt.*
4	quarts	1	gallon, *gal.*
36	gallons	1	barrel, *bbl. or bar.*
1½	barrels, or 54 gallons .	1	hogshead, *hhd.*

DRY MEASURE.

2 pints,	(*pt.*)	make 1 quart,	*qt.*
8 quarts	. . . . .	1 peck,	*pk.*
4 pecks	. . . . .	1 bushel,	*bu.*
8 bushels	. . . . .	1 quarter,	*qu.*
32 bushels, or 4 qrs.	. . .	1 chaldron,	*ch.*

A gallon, dry measure, contains 268.8 cubic inches.

Grain is generally sold by weight. By the laws of New York state, 60 pounds of wheat, 56 of rye or Indian corn, 48 of barley or 32 pounds of oats=1 bushel.

WINE MEASURE.

4 gills,	(*gi.*)	make 1 pint,	*pt.*
2 pints	. . .	1 quart,	*qt.*
4 quarts	. . .	1 gallon,	*gal.*
31½ gallons	. . .	1 barrel,	*bar.*
63 gallons	. . .	1 hogshead,	*hhd.*
2 hogsheads	. .	1 pipe,	*pi.*
2 pipes	. . .	1 ton,	*tun.*

TIME.

60 seconds,	(*sec.*)	make 1 minute,	*min.*
60 minutes	. . . . .	1 hour,	*hr.*
24 hours	. . . . .	1 day,	*d.*
7 days	. . . . .	1 week,	*wk.*
4 weeks	. . . . .	1 month,	*mo.*
12 calendar months, or 365 days 6 hours		1 civil year.	*yr.*

In computing interest, we say, 30 days make 1 month, 12 months 1 year.

The following lines express the number of days in each month:

Thirty days have September,
April, June, and November;
All the rest have thirty-one,
Excepting February alone,
To which we twenty-eight assign,
Till leap year gives it twenty-nine.

TABLE OF THE NAMES OF THE MONTHS AND THE NUMBER OF DAYS IN EACH.

1st	month	January	31
2d	"	February	28 (leap yr. 29.)
3d	"	March	31
4th	"	April	30
5th	"	May	31
6th	"	June	30
7th	"	July	31
8th	"	August	31
9th	"	September	30
10th	"	October	31
11th	"	November	30
12th	"	December	31

NEW YORK SHILLINGS AND PENCE IN FEDERAL MONEY.

6 pence	= 6¼ cents.	4*s.* 6*d.*	= 56¼ cents.
1 shilling	= 12½ "	5*s.*	= 62½ "
1*s.* 6*d.*	= 18¾ "	5*s.* 6*d.*	= 68¾ "
2*s.*	= 25 "	6*s.*	= 75 "
2*s.* 6*d.*	= 31¼ "	6*s.* 6*d.*	= 81¼ "
3*s.*	= 37½ "	7*s.*	= 87½ "
3*s.* 6*d.*	= 43¾ "	7*s.* 6*d.*	= 93¾ "
4*s.*	= 50 "	8*s.*	= 100 "

MISCELLANEOUS TABLE.

12 units . . .	make 1 dozen.
12 dozen	1 gross.
12 gross	1 great gross.
20 units	1 score.
56 pounds	1 firkin of butter.
100 pounds	1 quintal of fish.
200 pounds	1 barrel of pork, beef or fish.
196 pounds	1 barrel of flour.
24 sheets of paper . .	1 quire.
20 quires	1 ream.

A sheet folded in	2	leaves	forms a	folio.	
" "	4	"	"	quarto, or 4to.	
" "	8	"	"	octavo, or 8vo.	
" "	12	"	"	duodecimo, 12mo.	
" "	16	"	"	16mo.	
" "	18	"	"	18mo.	

An *aliquot part* of a given number is a number contained in that given number a whole number of times.

TABLE OF ALIQUOT PARTS.

Cents.	Parts of $1.	Parts of £1.	Parts of 1 shilling.
50	= $\frac{1}{2}$	10*s.* = $\frac{1}{2}$	6*d.* = $\frac{1}{2}$
33$\frac{1}{3}$	= $\frac{1}{3}$	6*s.* 8*d.* = $\frac{1}{3}$	4*d.* = $\frac{1}{3}$
25	= $\frac{1}{4}$	5*s.* = $\frac{1}{4}$	3*d.* = $\frac{1}{4}$
20	= $\frac{1}{5}$	4*s.* = $\frac{1}{5}$	2*d.* = $\frac{1}{6}$
16$\frac{2}{3}$	= $\frac{1}{6}$	3*s.* 4*d.* = $\frac{1}{6}$	1$\frac{1}{2}$*d* = $\frac{1}{8}$
12$\frac{1}{2}$	= $\frac{1}{8}$	2*s.* 6*d.* = $\frac{1}{8}$	1*d.* = $\frac{1}{12}$
6$\frac{1}{4}$	= $\frac{1}{16}$	1*s.* 8*d.* = $\frac{1}{12}$	
5	= $\frac{1}{20}$	1*s.* = $\frac{1}{20}$	

IN NEW ENGLAND.

$4\frac{1}{2}$ pence	=	$6\frac{1}{4}$ cents.
9 pence	=	$12\frac{1}{2}$ "
1*s.* 6*d.*	=	25 "
3*s.*	=	50 "
6*s.*	=	100 "

SECTION II.

REDUCTION.

1. In $4\frac{7}{16}$ pounds how many 16ths?

$4\times16=64$; $74\times7=71$. *Ans.* 71 sixteenths.

But, 16ths of a pound, are ounces, because 16oz.= 1 lb. Therefore, the method of reducing a whole or mixed number to an improper fraction, is also the method for reducing any denominate number to a lower denomination.

2. In £2 4s. 6d. how many pence?

Solution.—£2 4s.=44s. 44s. 6d.=534d. *Ans.*

3. In 534 pence how many pounds, and shillings.

Solution.—534d.=$\frac{534}{12}$s.=$44\frac{6}{12}$s.=44s. 6d.

44s.=$\frac{44}{20}$£. 2£. 4s. *Ans.* £2 4s. 6d.

This last example is like reducing an improper fraction to a whole or mixed number.

Reduction is the process of changing numbers from one denomination to another, without altering their value; as, 6 shillings=$6\times12=72$ pence=$72\times4=$ 288 farthings. 288 farthings=$288\div4=72$ pence= $72\div12=6$ shillings. Reduction from a *higher* to a

lower denomination is called *Reduction Descending*, and is performed by multiplication. Reducing from a lower to a higher denomination is *Reduction Ascending*, performed by division.

4. In £8 12s. how many shillings?
   In 172 shillings, how many pounds?
5. In 7lbs. 5oz. av., how many ounces?
   In 117 ounces, how many pounds, av.
6. In 15 cwt. 20lb., how many pounds?
   In 1520lb. how many hundred weight?
7. In 3 tons, 7 cwt., how many hundred weight?
   In 67 cwt., how many tons?
8. 9 rods, 10 feet, how many feet?
   In 158½ feet, how many rods?
9. In 25 yds., 2 ft., how many feet?
   In 77 feet, how many yards?
10. How many shillings are there in 4d., 6d., and 7d.? *Ans.* 1s. 5d.
11. How many shillings in 10d., 8d., and 5d.?
12. How many shillings will 3 pounds of butter cost at 1s. 5d. a pound? 3 times 5 pence is 15d.= 1s. 3d.? 3 times 5s. is 15s. to which add 1s., found in 15d., making 16s. *Ans* 16s. 3d.
13. What is the cost of 1 pound of butter at 1s. 6d., 1 pound of cheese at 9d., and 1 pound of raisins at 11d.?
14. What cost 4 yards of ribbon at 1s. 8d. a yard?
15. What cost 3lb. of beef at 10d. a pound, and 2lb. of ham at 1s. a pound?
16. What cost 8 yards of cotton cloth at 10d. a yard?
17. How many shillings cost 12½lb. of cheese, at 7d. a pound? 84d.+3½d.=how many shillings?

Or thus, 12lb. at 7d.=7lb. at 12d.=7s; and ½lb. at 7d.=3½d. *Ans.* 7s 3½d.

18. How many pounds in 12oz. 13oz. and 11lb. 6oz.? *Ans.* 2lb. 15oz.

19. Add, 8, 7, 5, 12, 10, and 14 ounces.

Thus; 8+7+5=1lb. 4oz.; 4+12=1lb., making 2lb.; 10+14=1lb. 8oz. *Ans.* 3lb. 8oz.

20. How many pounds in, 5, 6, 14, 9, 12, 8, and 7 ounces? Partial results, 1lb., 9oz.; 2lb., 2oz; 3lb. 6oz. *Ans.* 3lb. 6oz.

21. How much is 3s. 4d., 2s. 6d., and 5s. 7d.?

22. How many gallons in 3, 2, 1, 2 and 3 quarts?

23. If you buy, at different times, 1 gallon 2 quarts; 2 gallon 1 quart; 3 quarts, and 4 gallons 2 quarts of molasses, how much is there of the whole?

24. How many gallons of oil in 3 quarts 1 pint; 3 quarts, 2 quarts, 1 pint, and 2 quarts?

25. How much wine in 6 bottles, containing 1½ pint each?

SECTION III.

1. In New York currency, how many cents are there, in 6 pence? 1s.? 1s. 6d.? 2s.? 2s. 6d.? 3s.? 3s. 6d.? 4s.? 4s. 6d.? 5s.? 5s. 6d.? 6s.? 6s. 6d.? 7s.? 7s. 6d? 8s?

2. How many dollars in 8s.? 12s.? 16s.? 20s.? 24s.? 32s.? 36s.? 40s.?

Many merchants in New York keep their *prices* in shillings and pence; while they estimate the *cost* in Federal Money.

3. What cost 2 straw hats at 5s. each?

Ans. 10s.=$1.25.

4. What cost 8 yards of cloth at 5s. a yard. *Ans.* $5.

5. What cost $2\frac{1}{4}$ yards of edging at 2s. ?
" $1\frac{1}{4}$ " " at 5s.
and $1\frac{1}{2}$ yards of silk at 9s.
Ans. $3.03.

6. What cost 1 yard of Jean at 1s. 6d.; 9 yards of calico at 1s. 6d.; 6 yards of cambric at 1s. 4d.; and $2\frac{1}{2}$ yards of ribbon at 1s. ? *Ans.* $3.18\frac{3}{4}$.

7. What cost 5 yards of linen at 3s.; 1 yard of calico at 1s. 5d., and 3 yards of linen at 2s. 3d. ?
Ans. $2.90 nearly.

8. What cost 6 yards of cloth at 1s. 9d.; 6 yards at 1s. 6d.; $5\frac{1}{2}$ yards at 1s.; and 5 yards at 9d. ?
Ans. $3.59.

9. What cost 3 lbs. of yarn at 1s. 9d.; $1\frac{1}{2}$ yard of calico at 1s. 6d.; $\frac{1}{2}$ lb. tea at 5s.; and $\frac{1}{2}$ lb. at 3s. 6d. ?
Ans. 11s. 9d.=$1.47.

10. What cost 1 broom at 1s. 6d.; $\frac{1}{2}$ lb. tea at 3s.; and 9 yards of calico at 1s. 3d. ?
Ans. 14s. 3d.=$1.78.

11. What cost 1 gallon of molasses at 4s.; $\frac{1}{4}$ lb. of cassia at 2s. 6d.; and 2 lbs. of sugar at 1s. 1d. ?
Ans. 7s. 5d.=$0.93 nearly.

12. What cost 5 yards of ticking at 1s. 3d.; $\frac{1}{2}$ yard of flannel at 3s. 4d.; and 2 yards of muslin at 1s. 9d. ?
Ans. 11s. 5d.=$1.42, nearly.

13. What cost 7 lbs. of veal at 6d.; 4 lbs. of cheese at 9d.; and 2 lbs. of butter at 1s. 8d. ? *Ans.* $1.23.

14. What cost 3 yards of calico at 1s. 6d.; 10 yards of sheeting at 1s. 3d.; and 6 spools of thread at 4d. ? *Ans.* $2.37\frac{1}{2}$.

15. What cost 5 lbs. of raisins at 9d.; 3 lbs. of figs at 1s. 4d.; and 7 lbs. of sugar at 8d. ? *Ans.* $1.55.

CHAPTER VIII.

APPLICATIONS.

SECTION I.

GAIN AND LOSS.

1. If a merchant buys cheese at 7 cents a pound, how much does he gain or lose on 1 pound if he sells it at 8 cents? at 6 cents? at 5 cents? 9 cents? 4 cents? 10 cents? 11 cents?

2. If a man buys cheese at 7 cents a pound, his gain or loss will be how many times the cost, or what part of the cost, if he sells it at 8 cents? *Ans.* Gain $\frac{1}{7}$. What, if he sells it at 6 cents? *Ans.* Loss $\frac{1}{7}$. What, if he sells it at 9 cents? 5 cents? 10 cents? 4 cents? 11 cents? *Ans.* to the last two, Loss $\frac{3}{7}$. Gain $\frac{4}{7}$.

The answers in this last case show the RATE *of gain or loss. For convenience it is called the* RATIO. *It is equal to the gain or loss divided by the cost, and shows how much is gained or lost on* $1. Thus,

3. If a man buys 1 dollar's worth (or any other amount) of cheese at 7 cents a pound, how much does he gain or lose on $1, in selling it at 8 cents? 6 cents? 5 cents? 10 cents? 4 cents? 11 cents?

Ans. Gain $\$\frac{1}{7}$. Loss $\$\frac{1}{7}$, &c., as in example 2.

4. If a man buys cheese at 7 cents a pound, how many times, or what part of the cost, does he receive

for it in selling it at 8 cents? *Ans.* $1\frac{1}{7}$, or $\frac{8}{7}$. At 6 cents? *Ans.* $1-\frac{1}{7}$, or $\frac{6}{7}$.

At 9 cents? 10 cents? 5 cents? 4 cents? 11 cents?

Ans. $\frac{9}{7}$; $\frac{10}{7}$; $\frac{5}{7}$; $\frac{4}{7}$; $\frac{11}{7}$.

These last answers show the *amount of* $1 in each case.

The amount of $1 is equal to the avails divided by the cost. It is equal to 1 increased by the ratio of gain, or 1 diminished by the ratio of loss.

5. A man bought a hogshead of sugar for $45, and sold it for $50; what was the ratio of gain, or gain on $1?

First find the gain or loss by subtraction.

Ans. $\$\frac{5}{45}$, or $\$\frac{1}{9}$.

6. A merchant bought broad cloth at $2.25 a yard, and sold it at $3; what was the gain on $1?

Reduce the $3 *to* 300 *cents, because the other sum is* (225) *cents.*

$300-225=75$. $\frac{75}{225}=\frac{15}{45}=\frac{3}{9}=\frac{1}{3}$.

Ans. $\$\frac{1}{3}$, or $\$0.33\frac{1}{3}$.

7. A merchant bought cloth at $3 a yard, but it became damaged, and he sold it at $2.25; what was the loss on $1?

$\frac{75}{300}=\frac{25}{100}=\frac{1}{4}$. *Ans.* $\$\frac{1}{4}$, or $0.25.

8. A village which last year contained 2000 inhabitants, this year contains 2150; what is the ratio of increase for the year? *Ans.* $\frac{3}{40}$. *That is,* 3 *persons have been added to every* 40.

9. If the village contained 2150 persons last year, and only 2000 this year, what is the ratio of decrease? *Ans.* $\frac{3}{43}$.

10. A market woman commenced by buying $1.50 worth of fruit, and in 6 days she increased her money to $6; what was the ratio of gain for the 6 days?

Ans. 3. That is, the gain was 3 times the original stock.

11. In the last example, what was the amount of $1? *Ans.* 4. That is, the avails were equal to 4 times the stock.

12. A man, valuing his house at $2050, pays $25 a year to insure it against fire; what is the ratio of insurance? *Ans.* $\frac{1}{82}$.

In how many years would the insurance equal the value of the house?

13. A man borrowed $105, and at the end of a year paid $113 to discharge the debt; what ratio of interest did he pay? *Ans.* $\frac{8}{105}$. How much on $1?

Ans. 0.07\frac{7}{15}$.

The number of cents interest on $1 for 1 year is called the *rate per cent.*, because it is the same as the number of dollars on $100 for the same time. When the *denominator* of the ratio is 100, the *numerator* is the *rate per cent.*

14. A man paid $28.80 interest on $360 for a year; what was the rate per cent.? *Ans.* 8.

The ratio is $\frac{8}{25}$, which is reduced to $\frac{8}{100}$ by joining 0s to both terms, making $\frac{200}{2500}$, then dividing both by 25.

15. If the population of a town increases from 2600 to 2795 in 1 year, what is the rate per cent. of increase?

$$\frac{195}{2600} = \frac{7\frac{1}{2}}{100} \qquad \textit{Ans.}\ 7\tfrac{1}{2}.$$

16. A farmer stored 840 bushels of potatoes in autumn, but in spring he had only 480 bushels; what was the ratio of loss? *Ans.* $\frac{3}{7}$.

SECTION II.

When the ratio of gain or loss is given, to find the whole gain or loss, on a given sum or quantity, the solution only requires multiplication. For the gain on 2, is twice as much as the gain 1; on 3, 3 times as much; on $\frac{1}{2}$, $\frac{1}{2}$ as much, &c. So also the amount of 2, is twice as much as the amount of 1; that of $\frac{1}{2}$, $\frac{1}{2}$ as much, &c.

To find the amount, first find the gain, then add it.

1. A merchant paid \$1.12, \$1.60, and \$1.80 per yard for cloths; at what prices shall he sell them respectively, that his gain may be $\frac{1}{4}$; or 25 per. cent.?

Ans. \$1.40; \$2, and \$2.25.

2. For cloths which cost \$1.14, \$1.77, and \$2.07 per yard, what shall be the retail prices, to gain $\frac{1}{3}$ or $33\frac{1}{3}$ per cent.? *Ans.* \$152; \$2.36; \$2.76.

3. A man borrowed \$125.62, and at the end of 1 year paid $\frac{7}{100}$ or 7 per cent. for the use of it: how much was the interest? \$125.62=12562 cents. To multiply by $\frac{7}{100}$, multiply by 7, and divide by 100 by cutting off 2 figures. This gives

$879\frac{84}{100}$ cents=\$8.793. *Ans.*

4. What is the interest of \$56.12 for 1 year at 7 per cent., and what is the amount?

Ans. \$3.928; \$60.048.

5. What is the interest, and what the amount of \$75.25, for 1 year at 6 per cent.?

Ans. \$4.515; \$79.765.

6. At 7 per cent. for 1 year, what is the interest of \$7? \$7$\frac{1}{2}$? \$8? \$8$\frac{1}{2}$? \$10? \$15? \$17? \$35?

7. At 6 per cent. for 1 year, what is the interest of \$6? 7$\frac{1}{2}$? \$9? \$9$\frac{1}{2}$? \$18$\frac{1}{2}$? \$47? \$65? \$78?

8. At 8 per cent. for 1 year, what is the interest of \$6? \$7$\frac{1}{2}$? \$8$\frac{1}{2}$? \$9? \$9$\frac{1}{2}$? \$11$\frac{1}{2}$? \$12? \$12$\frac{1}{2}$?

SECTION III.

TO COMPUTE INTEREST FOR YEARS, MONTHS, AND DAYS.

When the interest of any sum is found for 1 year, we say that for 2 years it will be twice; for 3 years, 3 times as much; &c.; for half a year, $\frac{1}{2}$; for 1 month's $\frac{1}{12}$; for 2 months, $\frac{2}{12}$; &c.; for 1 day, $\frac{1}{30}$ of a month, interest, or $\frac{1}{360}$ of a year's interest; for 2 days, $\frac{2}{360}$; &c.

As, if 12 months=1 year, and 30 days=1 month, 360 days=1 year.

1. If the interest on a certain sum for 1 year is \$40.08, how much is it for 1 month? 2 months? 6 months? *Ans.* \$3.34; \$6.68; \$20.04.

2. If one year's interest is \$6.84, how much is it for 1 month? 3 months? 7 months? 11 months?
Ans. \$0.57; \$1.71; \$3.99; \$6.27.

3. If one year's interest is \$3.36, how much is it for 3 years, 5 months? *Ans.* \$11.48.

Find it for 3 years, then for 1 month, then for 5 months.

Otherwise thus, 3y., 5mo.=41mo.=$\frac{41}{12}$ years.

Then, \$3.36×$\frac{41}{12}$=\$11.48 by cancelling 12.

4. If one year's interest is \$10.80, how much is it for 5 months, 23 days? 5 mo.=150d. 150+23=173 days=$\frac{173}{360}$ year, \$10.80×$\frac{173}{360}$=3×173 cents, by cancellation=519. *Ans.* \$5.19.

5. If one year's interest is \$14.40, how much is it for 4 months, 1 day? *Ans.* \$4.84.

6. What is the interest of \$57.12 for 1 year, 7 months, at 7 per cent.? Int. for 1y.=\$3.998; 1mo.=\$0.333; 7mo.=\$2.332? *Ans.* \$6.33.

7. What is the interest of \$36.75 for 3 years, at 7 per cent.? *Ans.* \$7.717.

8. What is the interest of $39.77 for 9 months at 6 per cent.? *Ans.* $1.79 nearly.

9. What is the interest of $58.72 for 5 y. 5 mo. 5 d. at 7 per cent.? *Ans.* $22.32.

SECTION IV.

1. What number multiplied into 3×4 will produce 12? 24? 36? 1? $\frac{1}{2}$? $\frac{1}{3}$?

Ans. 1. 2. 3. $\frac{1}{12}$. $\frac{1}{24}$. $\frac{1}{36}$.

2. What is $12 \div 12$? $24 \div 12$? $36 \div 12$? $1 \div 12$? $\frac{1}{2} \div 12$? $\frac{1}{3} \div 12$?

3. What multiplied into $\frac{2}{3}$ of $\frac{6}{7}$ will produce $\frac{4}{7}$? 1? 2? $\frac{1}{2}$?

4. What multiplied into $\frac{3}{4}$ will produce $\frac{9}{10}$? $\frac{15}{21}$? $\frac{21}{33}$? $\frac{13}{40}$?

5. What multiplied into $2\frac{1}{2}$ will produce 1? 2? 3? $\frac{1}{3}$? $\frac{1}{8}$? $\frac{3}{4}$?

6. What is $1 \div \frac{5}{2}$? $2 \div \frac{5}{2}$? $3 \div \frac{5}{2}$?
$1 \times \frac{2}{5}$? $2 \times \frac{2}{5}$? $3 \times \frac{2}{5}$?

7. What is $\frac{1}{2} \times \frac{2}{3}$? $\frac{1}{3} \times \frac{1}{6}$? $\frac{3}{4} \times \frac{2}{5}$?

8. What multiplier multiplied into $\frac{3}{4}$ will produce 1? 2? 3? 4? 5? 6? 7? 8?

9. What multiplied into $\frac{3}{4}$ will produce $\frac{1}{2}$? $\frac{1}{8}$? $\frac{3}{4}$? $\frac{5}{7}$? $\frac{2}{5}$? $\frac{6}{11}$?

Each multiplier is $\frac{3}{4}$ of the multiplicand.

10. What multiplied into $2\frac{2}{3}$ will produce $1\frac{7}{8}$? 2? $2\frac{1}{2}$? $2\frac{2}{3}$? $3\frac{1}{3}$? 6?

Ans. $\frac{2}{3}$. $\frac{3}{4}$. $1\frac{1}{16}$. 1. $1\frac{1}{2}$. $2\frac{1}{4}$.

11. What multiplied into $3\frac{3}{7}$ will produce $3\frac{3}{7}$? 2? $4\frac{2}{7}$? $11\frac{1}{2}$? $2\frac{7}{8}$? $4\frac{4}{5}$?

Ans. $1\frac{1}{24}$. $\frac{14}{24}$. $\frac{20}{24}$. $3\frac{1}{2}$. $\frac{7}{8}$. $1\frac{2}{5}$.

12. What multiplied into $\frac{5}{6}$ of $1\frac{2}{3}$ will produce

$\frac{7}{8}$?	$\frac{7}{25}$?	5?	1?
Ans. $\frac{2}{3}$.	$\frac{6}{25}$.	$4\frac{2}{7}$.	$\frac{6}{7}$.

13. A man being asked how many sheep he had, said that $\frac{5}{8}$ of their number increased by 24 would make $1\frac{3}{8}$ times the number; what was the number of sheep?

$\frac{5}{8}$ No. $+24 = 1\frac{3}{8}$ No. Take $\frac{5}{8}$ No. from each.
$24 = \frac{6}{8}$ No.
24 is $\frac{6}{8}$ of what number? *Ans.* 32.

SECTION V.

1. Bought $\frac{2}{3}$ of a yard of cloth for 6 shillings; what would be the price of a yard at that rate?

$\frac{2}{3}$ yard costs 6 shillings;
1 yard costs how many shillings?

As many times as $\frac{2}{3}$ of a yard is contained in 1 yard, so many times 6 shillings must be the price, how many shillings? But $\frac{2}{3}$ is in 1, $\frac{3}{2}$ times. Therefore,

$\frac{3}{2}$ of 6 shillings = 9 shillings. *Ans.*

2. Bought $\frac{2}{7}$ of a hundred weight of cocoa for $11; what would 1 hundred weight cost at that rate?

3. If $\frac{2}{4}$ of a pound of cassia cost 18 cents, what is the price per pound?

4. If $\frac{2}{10}$ of a pound of starch cost 5 cents, what is the price per pound?

5. If $\frac{3}{5}$ of a hundred weight of sugar cost $6, what is the price of 1 hundred weight?

6. Peter has 72 cents, which is $\frac{4}{7}$ as much money as John has; how much has John?

7. If a boat sails 7 miles in $\frac{5}{6}$ of an hour, how far does it sail in 1 hour?

8. $\frac{5}{7}$ of the number of scholars in a certain school is 54; how many are in the school?

9. A man having sold $\frac{1}{3}$ and $\frac{1}{2}$ of his flock, had 60 sheep left; how many had he at first?

10. 14 is $\frac{2}{3}$ of what number?

11. 21 is $\frac{7}{8}$ of what number?

12. 16 is $\frac{8}{4}$ of what number?

13. 16 is $\frac{1}{8}$ of what number?

14. 17 is $1\frac{1}{8}$ of what number?

15. 18 is $1\frac{8}{7}$ of what number?

SECTION VI.

If $\frac{2}{3}$ of a yard of cloth costs 10 shillings, what will $\frac{3}{5}$ of a yard cost? *Ans.* 9 shillings.

$\frac{2}{3}$ of a yard costs 10 shillings;
$\frac{3}{5}$ of a yard costs how much?

As many times as $\frac{2}{3}$ of a yard is contained in $\frac{3}{5}$ of a yard, so many times 10 shillings will answer the question.

Therefore $\frac{3}{5} \times \frac{3}{2}$ of 10 shillings = 9 shillings by cancellation.

Or, 6 shillings $\div \frac{2}{3}$ = cost of 1 yard, which $\times \frac{3}{5}$ = cost of $\frac{3}{5}$ yards.

2. If $\frac{3}{4}$ of a barrel of flour costs a certain sum, what part of the same sum is $\frac{2}{3}$ of a barrel worth?

3. If a certain keg will hold $\frac{4}{7}$ of a barrel, what part of the same keg will contain $\frac{2}{3}$ of a barrel?

$\frac{4}{7}$ barrel = 1 keg;
$\frac{2}{3}$ barrel = how many kegs?

4. If $1\frac{1}{5}$ of a certain keg is equal to $\frac{3}{4}$ of a barrel, what part of a barrel will the keg contain?

$\frac{1}{3}$ keg=$\frac{2}{5}$ barrel;
1 keg=how much? *Ans.* $\frac{6}{5}$.

5. If $\frac{2}{3}$ of an acre of land cost $4, what costs $\frac{1}{3}$ of an acre? what costs 1 acre? 3 acres?

6. If $\frac{5}{6}$ of a ton of hay costs $10, what is the price per ton?

7. If $\frac{3}{4}$ of the distance that James walked is 9 miles, how far did he walk?

8. $2\frac{2}{3}$ is $\frac{4}{5}$ of what number? *Ans.* $3\frac{1}{3}$.

9. $2\frac{2}{3}$ is $1\frac{3}{5}$ times what number? *Ans.* $1\frac{2}{3}$.

10. $3\frac{3}{4}$ is $1\frac{2}{5}$ times what number?

11. $3\frac{3}{5}$ is $2\frac{2}{7}$ times what number?

12. $2\frac{5}{7}$ is $1\frac{5}{8}$ times what number?

13. $2\frac{5}{7}$ is $1\frac{7}{12}$ times what number?

14. $1\frac{7}{12}$ times $1\frac{5}{7}$ is what number?

SECTION VII.

QUESTIONS SOLVED BY ANALYSIS.

Observe that, in most questions for solution, one number is of the same name or kind with the answer, and the inquiry is, how many times that number, or what part of that number or quantity, is required to answer the question; as,

1. If 7 horses consume 3 tons of hay in 6 weeks, how many tons will 12 horses consume in 8 weeks?

Here 3 tons of hay is the term to be increased or diminished; how many times?

12 horses require $\frac{12}{7}$ as much as 7 horses;
11 weeks require $\frac{11}{6}$ as much as 6 weeks. Therefore
$\frac{12}{7}$ of 3 tons=hay for 12 horses for 6 weeks; and
$\frac{11}{6}$ of $\frac{12}{7}$ of 3 tons=hay for 12 horses for 11 weeks.
Cancel 6 and obtain $\frac{66}{7}$=$9\frac{3}{7}$ tons. *Answer.*

2. If a staff 4 feet high casts a shadow 7 feet long on level ground, how high is the steeple whose shadow at the same time measures 198 feet?

Solution.—How many times longer is the shadow of the steeple than that of the staff? $\frac{198}{7}$. Then it must be as $\frac{198}{7}$ times higher than the staff. *Ans.* $113\frac{1}{2}$ feet.

3. If 30 men can do a piece of work in 20 days, how many men can do the same in 8 days?

The less days, the more men. How many times the men for 20 days, are required for 8 days? $\frac{20}{8}$ times.
Ans. 75 men.

4. If 50 gallons of water in 1 hour fall into a cistern containing 230 gallons, and 35 gallons run out each hour, in what time will the cistern be filled?

How many gallons is the gain in 1 hour? 15 gallons. 15 is what part of 230 gallons? $\frac{15}{230}$, or $\frac{3}{46}$. Then $\frac{3}{46}$ filled in 1 hour; once filled in how many hours?
Ans. $15\frac{1}{3}$ hours.

5. A and B leave the same place, and travel the same road. A goes 5 days before B, 20 miles a day; B follows, going 25 miles a day. In what time and distance will he overtake A?

Solution.—A gets 100 miles the start.
If B gains $25-20=5$ miles in 1 day, in what time will he gain the 100 miles? *Ans.* —. Then how far will he go in that number of days? *Ans.* 500 miles.

6. If A can do a certain work in 3 days, and B in 4 days, in how many days can they do it together?

Solution.—What part will A do in 1 day? B in 1 day? A and B in 1 day? *Ans.* $\frac{1}{3}+\frac{1}{4}=\frac{7}{12}$. If $\frac{7}{12}$ is done in 1 day, in what time is 1 done? *Ans.* $1\frac{5}{7}$ days.

7. If A can do a work in 4 days, and B in 5 days, in what time can they both do it?

8. If A can do a work in 3 days, B in 4, and C in 5 days, in what time can the three do it?

Ans. $1\frac{3}{47}$ days.

9. A farmer being asked how many sheep he had, said, "If I had half as many, $\frac{1}{3}$ as many, and $\frac{1}{5}$ a sheep, in addition to my flock, I should have 50." How many had he?

Solution.—Taking $\frac{1}{2}$ from 50, leaves $49\frac{1}{2}=\frac{99}{2}$.
Then $\frac{99}{2}$ is 1 and $\frac{1}{2}$ and $\frac{1}{3}$ times what number?

Ans. 27.

10. If when wheat is \$1.10 a bushel, rye is 77 cents, what should be the price of rye when wheat is \$1.50 cents? *Ans.* \$1.05.

11. If 7 men eat 56 lbs. of bread in 16 days, how many pounds will 21 men eat in 6 days?

Ans. 63 lbs.

SECTION VIII.

1. A gentleman paid a certain sum for a village lot, built a house which cost $2\frac{1}{2}$ times as much, and a fence which cost \$50. The whole expense was \$50 less than $3\frac{3}{4}$ times the cost of the lot. How much was its cost?

Solution—Add \$50 to the expense, and the result is,
The whole cost$=3\frac{3}{4}$ times that of the lot.
Then, once a number$+2\frac{1}{2}$ times the same$+100=3\frac{3}{4}$ times the same number.
Now take away $3\frac{1}{2}$ times the number, and there is left $100=\frac{1}{4}$ the number. *Ans.* \$400.

2. A man has a horse, a harness worth $\frac{1}{4}$ as much as the horse, and a wagon worth \$70. The three are

worth $50 less than twice the value of the horse. How much is the horse worth? *Ans.* $150.

3. A farmer being asked how many cows he had, said, "If I should buy half as many, and one third as many as I now have, and then sell $2\frac{1}{2}$ cows, I should have 25 left." How many had he?

Solution.—*If he should not sell the* $2\frac{1}{2}$, *he would have how many?* $27\frac{1}{2}=(1+\frac{1}{2}+\frac{1}{3})$ *times what number?* *Ans.* 15.

4. $\frac{1}{3}$ and $\frac{1}{3}$ of a certain number make $\frac{2}{3}$ of a unit more than $\frac{1}{2}$ of the number; what is the number?

5. $\frac{1}{3}$ and $\frac{1}{3}$ of a certain number make $1\frac{2}{3}$ units less than $\frac{2}{3}$ of the number; what is the number?

6. $\frac{1}{4}$ and $\frac{1}{7}$ of a number make $\frac{5}{7}$ of a unit more than $\frac{1}{3}$ of the number; what is the number?

Answer to the last three, 12.

7. Buying a hat, coat, and cloak, the hat cost $4; the coat as much as the hat and half of the cloak; and the cloak as much as the hat and coat: what was the whole cost? *Ans.* $32.

8. Peter and John bought a bushel of apples, for which Peter paid 20 cents, and John 25 cents; what part of the apples should each have?

Solution.—20+25=45 cents=whole cost; therefore Peter should have $\frac{20}{45}$, and John $\frac{25}{45}$. *If they sold the apples for* 90 *cents, how many cents should each have?* $\frac{20}{45}=\frac{4}{9}$. $\frac{4}{9}$ of 90 cents=40 cents, Peter's share; 50 cents, John's.

9. A and B built a wall together. A worked 7 days, and B 5 days. They received $30 for the job; how much of the money belongs to each?

Ans. A, $17.50; B, $12.50.

10. A had 3 cows pastured for 4 weeks; B had 2 cows pastured for 5 weeks in the same field. They

both paid $8.80 cents. How much should each pay?

A had $8\times4=12$ weeks; B had $2\times5=10$ weeks' pasturing. Sum=22 weeks. A=$\frac{12}{22}$; B=$\frac{10}{22}$; or, A=$\frac{6}{11}$, B=$\frac{5}{11}$. *Ans.* A $4.80; B $4.

11. A, B. and C did a piece of work, in which A worked 10 days, B 12, and C 18. They received $50: how much of it belongs to each?

Ans. A $12.50; B $15; C $22.50.

12. A and B bought a span of horses, for which A paid $100, and B $131. They sold the horses for $264. How much of the money is A's? *Ans.* 114\frac{2}{7}$.

13. A, B, and C built 80 rods of wall, for which they received $176. A built 20 rods, B 25, and C 35. How much should each receive?

Ans. A $44; B 55; C $77.

14. If A's labor is worth $1.50, and B's $1 a day, and they together do a job in 20 days, for which they are paid $60, how much shall each receive?

Ans. A $36; B $24.

15. A sent 3 men for 10 days, at $1.50 a day; B, 2 men for 12 days, at $1.25 a day each, to do a job, for which they received $100. What is each one's share?

Ans. A's $60; B's $40.

SECTION IX.

HERE REVIEW SECTION XIII.—P. 121.

1. If $5\frac{1}{2}$ yards of cloth cost $3\frac{4}{5}$, what costs 1 yard? *Ans.* $\frac{2}{11}$ of $\frac{4}{5}$ dollars.

If 1 *yard cost* $\frac{2}{11}$ *of* $\frac{4}{5}$ *dollars, what cost* $3\frac{3}{5}$ *yards?*

Ans. $\frac{1}{5}$ *of* $\frac{2}{11}$ *of* $\frac{4}{5}$ *dollars, or,* $2\frac{8}{15}$ *dollars.*

2. If $2\frac{3}{8}$ yards, cost $\$4\frac{1}{2}$, what costs $\frac{5}{8}$ of a yard?

Ans. $\$1\frac{1}{4}$.

3. How many yards of cloth that is 3 quarters wide, are equal to 50 yards, 5 quarters wide?

Ans. $83\frac{1}{3}$ yards.

4. How many feet in length of a pile of wood 4 feet wide, and 6 feet high, will fill a wood-house 20 feet wide, 12 feet high, and 40 feet long?

Ans. 400 feet.

5. How many pages of a book having 42 lines on each page, and 48 letters in each line, are equal to 160 pages having 34 lines on each page, and 42 letters in each line? *Ans.* $113\frac{1}{3}$.

6. Peter and John run a race, in which Peter gains 3 rods in running 11 rods; how far must he run to gain 7 rods?

As many times as 3 *rods of gain are contained in* 7 *rods of gain, so many times* 11 *rods must be contained in the answer.*

$\frac{7}{3} \times 11 = \frac{77}{3} = 25\frac{2}{3}$ *rods. Ans.*

7, A ship sailing 10 miles an hour, is 6 miles ahead of a steamer moving 15 miles an hour; how far must the steamer go to overtake the ship?

Ans. 18 miles.

8. Suppose a fox that runs 130 rods a minute, has 40 rods the start of a dog that can run 160 rods a minute; how long and how far must the dog pursue the fox, to overtake him?

Ans. $213\frac{1}{3}$ rods, in $1\frac{1}{3}$ minutes.

9. A cistern containing 200 gallons, is fed by a pipe which lets in 4 gallons of water in 5 minutes, and has a faucet which will let out 6 gallons in 7 minutes. If when the cistern is full, the faucet is opened, in what time will the cistern be emptied; and

how much water will be poured in, and how much drawn out in that time?

Solution.—$\frac{3}{7}$ *gallon drawn, and* $\frac{4}{5}$ *gallon let in per minute.*

$\frac{6}{7}-\frac{4}{5}=\frac{2}{35}$ *gallon net loss in* 1 *minute.*

$200\div\frac{2}{35}$, or, $100\times35=$*number of minutes.*

Ans. 58 *hours* 20 *minutes.*

$\frac{4}{5}$ *of* 3500=2800 *gallons let in.*

$\frac{6}{7}$ *of* 3500=3000 *gallons drawn out.*

3000—2800=200 *gallons, contents of the cistern.*

10. A merchant bought cloth at the rate of \$4 for 5 yards, and sold it at the rate of \$6 for 7 yards, and gained \$200. How many yards were there?

11. A merchant bought 3500 yards of cloth, at the rate of \$4 for 5 yards, and gained \$200 in the sale of it. How much per yard did he receive for it?

Ans. \$$\frac{6}{7}$.

12. A can do a certain work in 5 days; B can do the same in 4 days; in how many days can they do it together?

Solution.—A does $\frac{1}{5}$, *B* $\frac{1}{4}$ *in* 1 *day,* $\frac{1}{5}+\frac{1}{4}=\frac{9}{20}$ *done by both in* 1 *day. Then,*

$\frac{9}{20}$ *of the job requires* 1 *day;*

1 *job requires how many days?*

As many as the number of times that $\frac{9}{20}$ *is contained in* 1, *so many times* 1 *day will answer the question.*

Ans. $\frac{20}{9}=2\frac{2}{9}$ *days.*

13. If A can do a certain work in 3 days, B in 4 days, and C in 5 days, in how many days, can the three do it? *Ans.* $1\frac{3}{47}$ days.

14. If A, B, and C can do a certain work in $1\frac{13}{47}$ days, and A can do it in 3 days, and B in 4 days, in how many days can C do it alone?

They all do it in $\frac{60}{47}$ *days, which is* $\frac{47}{60}$ *of it in* 1 *day,* $\frac{47}{60}-\frac{1}{3}-\frac{1}{4}=\frac{13}{60}=\frac{1}{5}$, *which C does in* 1 *day.*

15. When after 1 o'clock is the minute hand of a atch, exactly with the hour hand?

Ans. At $1\frac{1}{11}$ hour, or, 1 h. $5\frac{5}{11}$ min.

16. If a staff 4 feet high, casts a shadow $6\frac{1}{2}$ feet ng, how high is the steeple which casts a shadow $9\frac{1}{2}$ feet at the same time? *Ans.* 92 feet.

17. If $30\frac{1}{2}$ yards of cloth cost \$27,50, what will) pieces cost, each piece containing $30\frac{1}{2}$ yards?

18. How many rolls of paper, 2 feet wide, and 9 ards long, will cover the sides and ends of a room 24 et long, 18 feet wide, and 9 feet high?

This solution is reduced to $(42\times2\times9)\div(2\times9\times$. *Cancel* $2\times9\times3$. *Ans.* 14 pieces.

19. How many shingles, each covering a surface 4 ches by 6, will cover both sides of a roof 40 feet ng, having the rafters 24 feet long? *Ans.* 11520.

20. If $37\frac{1}{2}$lb. of sugar cost $281\frac{1}{4}$ cents, what will .b. cost? 10lb.? 50lb.?

Ans. $7\frac{1}{2}$ cents; 75 cents; 375 cents.

21. If $9\frac{3}{4}$ tons of coal cost $\$50\frac{7}{8}$, what is the price er ton? the cost of 20 tons? *Ans.* $\$6\frac{1}{2}$; \$130.

SECTION X.

1. What part of £1 is 12s. 6d.?

SOLUTION.

$$\begin{array}{l|l} 6\times2=12 & 6\text{d.}=\frac{1}{2}\text{s.} \\ 5\times4=20 & 12\frac{1}{2}\text{s.}=\frac{5}{8}\text{s.} \\ & \frac{5}{8}£. \ \textit{Ans.} \end{array}$$

Divide the least denomination by such divisor as ill reduce it to the next greater; add the quotient

(*fraction*) *to that next greater; divide this mixed number by such divisor as will reduce it to the next higher, and so on, to the denomination required.*

2. In £$\frac{5}{8}$, how many shillings, pence, &c.?

$$\begin{array}{r|l} 8)5 & \\ 20 & \\ \hline 100 & (12\text{s. } 6\text{d.} \\ 96 & \\ \hline 4 & \\ 12 & \\ \hline 48 & \end{array}$$

£$\frac{5}{8}$ = £5 ÷ 8 = 100s. ÷ 8 = 12s. and 4s. = 48d. rem. 48d. ÷ 8 = 6d. *Ans.* 12s. 6d. That is, *Reduce the numerator to the next lower denomination, and divide by the denominator. If there is a remainder, reduce it to the next lower denomination, and divide as before.*

3. 4 oz. 16 pwt. is what part of 1 lb. Troy?

What is the value of $\frac{3}{4}$ lb. Troy?

4. 3 w. 1 d. 9 h. 36 min. is what part of a month of 4 weeks?

In $\frac{1}{5}$ mo., how many days, hours, and minutes?

5. Reduce 2 R. 26$\frac{2}{3}$ sq. r. to the fraction of an acre.

In $\frac{2}{3}$ of an acre, how many roods and rods?

6. Reduce 5s. 6d. New York currency to the fraction of a dollar.

In $\frac{1}{11}$ of a dollar, how many shillings and pence?

7. Reduce 2 mo. 10 d. to the fraction of a year.

In $\frac{7}{8}$ of a year, how many months and days?

8. Reduce 1 foot 2 in. to the fraction of a yard.

In $\frac{7}{12}$ yd., how many feet and inches?

9. If 2 roods 16 rods of land cost $80, what will $\frac{3}{4}$ of an acre cost?

If $\frac{3}{4}$ of an acre cost $100, what will 2 roods 15 rods cost?

10. If 6d. New York, is equal to 4$\frac{1}{2}$d. New England, what is the value in New England of 7s. New York?

If 5s. 3d. New England is equal to 7s. New York, what is the value in New York of $4\frac{1}{2}$d. New England?

11. If a merchant buys calico in New York at 8d. a yard, at what price must he sell it in Vermont, to make a profit equal to $\frac{1}{4}$ of the cost? *Ans.* $7\frac{1}{2}$d.

12. If James is 12 years old, and $\frac{3}{4}$ of his age is equal to $\frac{3}{5}$ of John's age, how old is John?

Ans. 15 years.

13. A man being asked how many sheep he had, said that he had them in two pastures; in one pasture he had 25; and that $\frac{5}{7}$ of their number was equal to $\frac{4}{7}$ of the number in the other pasture: how many sheep had he? *Ans.* 40.

14. It is required to put 15 gals. 3 qts. of wine into jugs containing 1 qt. 2 qts., and 1 gal. 3 qts. 1 pt. each; using an equal number of each kind. How many of each are required? *Ans.* 6.

15. B is twice as old as A; C is $2\frac{1}{2}$ times as old as both A and B; and the sum of their three ages is 84 years: what is the age of each?

Ans. A 8; B 16; C 60 years.

16. If A earns \$2 a day, B \$1.75, and C \$1.50, in how many days will the three together earn \$34.125?

Ans. $6\frac{1}{2}$ days.

17. Four farms are such that A's contains 4 acres, as many times as B's contains 5 acres, C's 6, and D's 7 acres, and the four farms contain 660 acres; how many acres are there in each?

Ans. 120, 150, 180, and 210 acres.

18. If 8 horses eat 15 bushels of oats in 3 days, how many bushels will 16 horses eat in 9 days?

16 *horses will eat* ($\frac{16}{8}$) *twice as much as* 8 *horses, and* 9 *days will require* ($\frac{9}{3}$) 3 *times as much as* 3 *days.* $15 \times \frac{16}{8} \times \frac{9}{3} = 90$ *bushels. Ans.*

19. If 5 men build 54 rods of wall in 9 days, working 8 hours a day, how many rods will 7 men build in 20 days, working 10 hours a day?

Ans. 210 rods.

20. If 5 men build 54 rods of wall in 9 days, working 8 hours a day, how many hours a day must 7 men work to build 210 rods in 20 days?

Ans. 10 hours.

21. If 7 men build 210 rods of wall in 20 days, working 10 hours a day, in how many days will 5 men build 54 rods, working 8 hours a day?

Ans. 9 days.

Let the pupil make four more similar changes in question 19*th.*

22. If 65 men earn $5931.25 in 73 days, how much will 37 men earn in 97 days?

Ans. $4486.25.

23. If the interest of $100 for 12 months, is $7, what is the interest of $225 for 20 months?

Ans. $26.25.

24. If $3\frac{1}{5}$ yards of cloth cost $4, what will $8\frac{2}{5}$ yards cost? *Ans.* $10.50.

25. If $2\frac{2}{3}$ bushels of wheat cost $3\frac{2}{3}$, what will $6\frac{2}{3}$ bushels cost? *Ans.* $8.80.

26. A man owing $\frac{4}{5}$ of a vessel, sold $\frac{3}{5}$ of his share for $1000; what was the vessel worth? *Ans.* $1875.

27. A has a sum of money which being increased by its $\frac{1}{2}$ and $\frac{1}{8}$, amounts to $396; and $\frac{1}{2}$ of A's money is equal to $\frac{1}{5}$ of B's; how much has B?

Ans. $540.

28. A, B, and C, have $330; B's money is equal to $1\frac{1}{4}$ A's; and C has $50 more than B; how much has A? *Ans.* $80.

Solution.—330—50=280, *the number of dollars which*

they would all have, if C had no more than B. In that case, B's and C's would be equal to $2 \times 1\frac{1}{4}$ *A's* $= \frac{5}{2}$ *A's. Now as often as A has* 1, *the other two have* $\frac{5}{2}$, *and all have* $1\frac{5}{2}$ *or* $\frac{7}{2}$. *Then* 280 *is* $\frac{7}{2}$ *of what number?*

Ans. $\frac{2}{7}$ of 280 = 80.

29. D, E, and F, have \$420; E's money is equal to $1\frac{1}{5}$ D's; and F has \$80 more than E; how much has D? *Ans.* \$100.

30. Three pieces of cloth contain 95 yards; the first piece is $\frac{4}{5}$ of the second, and the third contains 20 yards less than the second; how many yards in each piece? *Ans.* 35, 40, 20.

31. Divide 197, into four parts such that the second part shall be 5 greater than the first; the third 7 greater than the second; and the fourth 8 greater than the third?

Solution.—Set aside all the inequalities by which each greater exceeds the least; which are

5 *for the second* = 5.
7 *more than* 5 *for the third* = 12.
8 *more than* 12 *for the fourth* = 20.
The sum of which = 37.
197 − 37 = 160; 160 ÷ 4 = 40, *the least part.*

Ans. 40, 45, 52, 60.

32. Two pieces of cloth contain 75 yards; and one piece is 10 yards longer than the other; what is the length of each?

33. Three pieces of cloth contain 92 yards; the second piece is 5 yards longer than the first; and the third is 7 yards longer than the second; what is the length of each? *Ans.* 25, 30, 37 yards.

THE END.

Our reasonings on all the higher principles of arithmetic are, in their *nature*, algebraic; no less so when we express the terms employed, by cumbrous words and phrases, than when we avail ourselves of the short hand which algebra furnishes.

The Publishers, in offering these Books to the public, are confident that they are superior in many respects to any other works on the same subject now in use. From the many notices of these works from teachers and others who have examined them, we present the following

RECOMMENDATIONS.

From Wm. H. Allen, L. L. D., President of Girard College.

In a letter to the Author, dated Feb. 14, 1853, President Allen says:—

"I have examined your High School Arithmetic with much interest, and am pleased with the simplicity and clearness of its methods. I consider it a valuable addition to our means of instruction in the important branch of which it treats. WM. H. ALLEN."

From O. P. Williams, Principal of the Union School, Little Falls, N. Y.

MESSRS. CRANDALL & MOSELEY,—

GENTS:—Blanchard's Arithmetic has been used nearly a year in the School under my charge, and, during that time, has been studied by more than one hundred different pupils, from eight to sixteen years of age, with the most gratifying result. None of them, not even the younger classes, have experienced any difficulty in readily comprehending and applying the rules, and formulas in Percentage, while the examples are sufficiently intellectual, without being severe or repulsive. The work possesses excellencies not to be found in any other book in use. Yours truly,

O. P. WILLIAMS.

LITTLE FALLS, Oct. 15, 1853.

From F. H. Weld, Principal of Utica Free Academy.

Mr. O. Blanchard,—

Sir,—About one year since, your High School Arithmetic was introduced as a text-book into the Utica Free Academy. At the present time we have over one hundred pupils using the work. It gives me great pleasure to say, that it answers our highest expectations, and we can with great confidence recommend its extensive use in our schools.

Yours truly, F. H. WELD.

Utica, Oct. 5, 1853.

From Richard Ellis, Professor of Mathematics in Oneida Conference Seminary.

I have used the High School Arithmetic of Prof. Orlando Blanchard, of Cazenovia, as a text-book thus far during the present term. I am prepared to say that, as a whole, I consider it equal to any work on that subject, for the purpose of instruction in Higher Schools. The subjects of Percentage, the extraction of the Cube Root, the Properties of Numbers, and the Tests of Divisibility, are made clear and interesting. RICHARD ELLIS.

Cazenovia, Feb. 19, 1853.

From the Committee on Text-books for the Union School, Cangstota.

Messrs. Crandall & Moseley,—

Gents:—We have examined and adopted in our School, Prof. Orlando Blanchard's Arithmetic published by you. It is, in our opinion, a work of superior merit. The arrangement of the subjects, the simple yet conclusive demonstrations, the truly practical and business-like problems which follow the examples under each rule, afford to the student facilities which cannot fail, under competent instruction, to make him a practical arithmetician. We are convinced of its superiority, as an elementary treatise, to any now in use.

CHARLES A. SPENCER, L. FOWLER,
W. STICKNEY, F. T. DRAKE.

HARTWICK, Oct. 17, 1853.

I intend to introduce Blanchard's Arithmetic into my School, and also to do what I can to have it introduced into the District Schools, in this and the adjoining Districts. I consider it to be far in advance of any other work that I have examined, in presenting the principles of Arithmetic in a more concise and clear form, and in adapting the demonstrations to the capacity of any scholar who is old enough to commence the study. The use that is made of Algebra in this work, I like much, and the author's manner of proving the "Tests of Divisibility," is the best that I have seen; and I would make the same remark of his method of treating the Cube Root.

GEO. P. PROCTOR.

WARNERVILLE, Oct. 17, 1853.

The Faculty of the Empire Union Seminary have examined Blanchard's High School Arithmetic, and are so well convinced of its merits, that we have adopted it as the text book on that branch of science in this Institution.

H. BURCHARD, Principal.

CAZENOVIA, Dec. 8, 1852.

I have examined Professor O. Blanchard's "High School Arithmetic," and consider it the very best introduction to the Higher Mathematics that I have seen.

JOHN W. ARMSTRONG,
Professor of Experimental Science.

From Rev. Joseph E. King, A. M., Principal of Fort Plain Seminary, and Female Collegiate Institute. Late Principal of the Collegiate Institute, at Newbury, Vt.

PROF. BLANCHARD,—

DEAR SIR,—For higher classes, those designing to teach, and also as a preparation for the study of the Higher Mathematics, I know of nothing equal to your High School Arithmetic. We shall take great pleasure in introducing it at Fort Plain Seminary. Respectfully yours,

JOSEPH E. KING.

NEWBURY, Vt., Oct. 29, 1853.

From Professors Jackson and White, of the Oneida Conference Seminary, Cazenovia, N. Y.

From actual experience in the recitation room, it affords us pleasure to offer our testimony to the superior merit of Blanchard's Arithmetic. The book seems to possess the rare quality of being destitute of everything that can be called lumber; and yet there is much original matter, much original reasoning, and, what is of more value than all questions about originality, in an elementary treatise, there is a perspicuity, a unity of method prevailing in all its parts, that renders it more than any book we have seen, peculiarly adapted to instruction.

GEO. JACKSON.
AARON WHITE.

From the Teachers of Mathematics in Fairfield Academy.

Having used Blanchard's Arithmetic as a Text-book, we are prepared to give it a high place among the books on that important branch of Mathematics. It is alike adapted to the wants of the student and the man of business. In it the "Theoretical and Practical" are most happily united. The inductive process, adopted by the author, leads the pupil to a ready perception of the principles upon which the rules are founded, and the examples, though not tedious, are, we think, sufficient and well selected. The manner in which the author has treated the subjects of Percentage, Cube Root, the Progressions, and his Demonstrations of many interesting properties of Numbers, are a *few* of the *many* merits of the work, which will render it acceptable to the advanced student. We know of no other work so well calculated to prepare the pupil for the study of the Higher Mathematics, as this. We hope to see it generally introduced into the schools of this country.

B. BARKER,
J. B. HOLCOMB.

FAIRFIELD, Sept. 17, 1853.

On the ground of its general merits, I have introduced it into my school, and believe it to be a work which will commend itself to the attention of teachers. A. PHELPS.

CARBONDALE, Aug. 22, 1853.

RICHBORO ACADEMY, June 7th, 1858.

Blanchard's High School Arithmetic appears to me to be greatly in advance of other works on Arithmetic that have yet been published. It bears internal evidence of having been written by one who has had extensive experience in the duties of the recitation-room, and who knows the wants of the pupil and the teacher. The author's ample illustration of the principles of Percentage, and their application to all cases that can be conceived, and his complete demonstration of the best and shortest method of extracting the Cube Root, leave little to be attempted by way of improvement, upon those subjects at least. I have introduced it into this institution, and we shall probably make it, and the Primary Arithmetic by the same author, our only text-books on that subject.

J. A. BADGER, Principal.

MORRISVILLE UNION SCHOOL, June 6, 1853.

I have, for some months past, used Blanchard's High School Arithmetic, as a text-book in this school, and purpose to use no other, because I believe it to be the best arithmetic yet published, being not only, as I think, natural in its arrangement, but concise and explicit in its definitions, and rigidly exact in its demonstrations. I am particularly pleased with his manner of treating the Roots, and the Properties of Numbers. Everything in this book has its *proof;* which cannot be said of all its predecessors.

A. H. FORT, Principal.

☞ The books are furnished at reduced rates for introduction, and Teachers wishing copies for examination can receive them free of charge, and postage pre-paid, by transmitting fifteen cents in postage-stamps.

These works are also sold at publisher's prices, wholesale and retail, by LEAVITT & ALLEN, 27 *Dey Street, New York.*

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